Scrivere in funzione di $t = \tan \frac{\alpha}{2}$

265 $\dfrac{2\sin\alpha + 4\cos\alpha}{\cos\alpha} - 2\tan\alpha =$ [4]

$$= \frac{2\dfrac{2t}{1+t^2} + 4\dfrac{1-t^2}{1+t^2}}{\dfrac{1-t^2}{1+t^2}} - 2\frac{2t}{1-t^2} =$$

$$= \frac{\dfrac{4t+4-4t^2}{\cancel{1+t^2}}}{\dfrac{1-t^2}{\cancel{1+t^2}}} - \frac{4t}{1-t^2} =$$

$$= \frac{4t+4-4t^2}{1-t^2} - \frac{4t}{1-t^2} = \frac{\cancel{4t}+4-4t^2-\cancel{4t}}{1-t^2} = \frac{4(\cancel{1-t^2})}{\cancel{1-t^2}} = 4$$

333 $\dfrac{\cos 2\alpha + \sin 2\alpha}{2} + \dfrac{\sqrt{2}\sin\left(2\alpha - \frac{\pi}{4}\right)}{2\cos 4\alpha} =$ $\left[\dfrac{\sin 2\alpha \cos 2\alpha}{\cos 2\alpha + \sin 2\alpha}\right]$

$2\alpha = \beta$

$$= \frac{\cos\beta + \sin\beta}{2} + \frac{\sqrt{2}\sin\left(\beta - \frac{\pi}{4}\right)}{2\cos 2\beta} =$$

$$= \frac{(\cos^2\beta - \sin^2\beta)(\cos\beta + \sin\beta) + \sqrt{2}\left[\sin\beta \overbrace{\cos\frac{\pi}{4}}^{\frac{\sqrt{2}}{2}} - \cos\beta \overbrace{\sin\frac{\pi}{4}}^{\frac{\sqrt{2}}{2}}\right]}{2(\cos^2\beta - \sin^2\beta)} =$$

$$= \frac{(\cos^2\beta - \sin^2\beta)(\cos\beta + \sin\beta) + \sin\beta - \cos\beta}{2(\cos^2\beta - \sin^2\beta)} =$$

$$= \frac{(\cos\beta - \sin\beta)(\cos\beta + \sin\beta)^2 - (\cos\beta - \sin\beta)}{2(\cos\beta - \sin\beta)(\cos\beta + \sin\beta)} =$$

$$= \frac{(\cos\beta - \sin\beta)(\cos\beta + \sin\beta)^2 - (\cos\beta - \sin\beta)}{2(\cos\beta - \sin\beta)(\cos\beta + \sin\beta)} =$$

$$= \frac{\cancel{(\cos\beta - \sin\beta)}\left[(\cos\beta + \sin\beta)^2 - 1\right]}{2\cancel{(\cos\beta - \sin\beta)}(\cos\beta + \sin\beta)} =$$

$$= \frac{\overbrace{\cancel{\cos^2\beta + \sin^2\beta}}^{1} + 2\cos\beta\sin\beta \cancel{- 1}}{2(\cos\beta + \sin\beta)} =$$

$$= \frac{\cancel{2}\cos\beta\sin\beta}{\cancel{2}(\cos\beta + \sin\beta)} = \frac{\cos 2\alpha \ \sin 2\alpha}{\cos 2\alpha + \sin 2\alpha}$$

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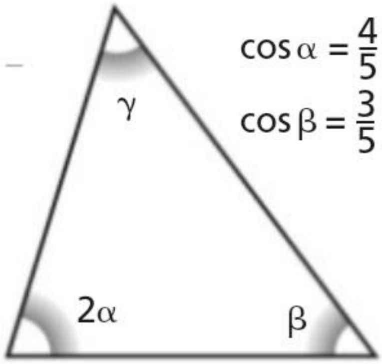

$\cos\alpha = \frac{4}{5}$

$\cos\beta = \frac{3}{5}$

Calcola sin γ e cos γ.

$\left[\frac{4}{5}; \frac{3}{5}\right]$

$$\gamma + 2\alpha + \beta = \pi$$

$$\gamma = \pi - 2\alpha - \beta = \pi - (2\alpha + \beta)$$

$$\sin\gamma = \sin(\pi - (2\alpha + \beta)) =$$

$$= \sin(2\alpha + \beta) =$$

$$= \sin 2\alpha \cos\beta + \cos 2\alpha \sin\beta =$$

$$= 2\sin\alpha\cos\alpha\cos\beta + (2\cos^2\alpha - 1)\sin\beta =$$

$$= 2\sqrt{1-\cos^2\alpha}\cdot\cos\alpha\cdot\cos\beta + (2\cos^2\alpha - 1)\cdot\sqrt{1-\cos^2\beta} =$$

$$= 2\sqrt{1-\frac{16}{25}}\cdot\frac{4}{5}\cdot\frac{3}{5} + \left(2\cdot\frac{16}{25} - 1\right)\cdot\sqrt{1-\frac{9}{25}} =$$

$$= 2\cdot\frac{3}{5}\cdot\frac{4}{5}\cdot\frac{3}{5} + \frac{7}{25}\cdot\frac{4}{5} = \frac{72}{125} + \frac{28}{125} = \frac{100}{125} = \frac{4}{5}$$

$$\cos\gamma = \sqrt{1-\sin^2\gamma} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

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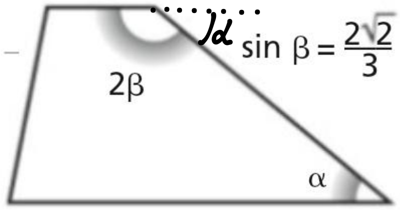

Calcola $\cot\alpha$.

$\left[\frac{7\sqrt{2}}{8}\right]$

$$\alpha + 2\beta = \pi$$

$$\alpha = \pi - 2\beta$$

$$\sin\alpha = \sin(\pi - 2\beta) = \sin 2\beta =$$

$$= 2\sin\beta\cos\beta =$$

$$= 2\sin\beta\sqrt{1-\sin^2\beta} =$$

$$= 2\,\frac{2\sqrt{2}}{3}\sqrt{1-\frac{8}{9}} = 2\cdot\frac{2\sqrt{2}}{3}\cdot\frac{1}{3} =$$

$$= \frac{4\sqrt{2}}{9}$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\sqrt{1-\sin^2\alpha}}{\sin\alpha} =$$

$$= \frac{\sqrt{1-\frac{32}{81}}}{\frac{4\sqrt{2}}{9}} = \frac{\sqrt{\frac{49}{81}}}{\frac{4\sqrt{2}}{9}} = \frac{\frac{7}{\cancel{9}}}{\frac{4\sqrt{2}}{\cancel{9}}} = \frac{7}{4\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{8}$$