1 Calcola il valore di: **a.** $\sin\frac{\pi}{8} + \cos\frac{\pi}{12}$; **b.** $\sin 72° \cos 18° + \cos 72° \sin 18°$.

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a) $\sin\frac{\pi}{8} + \cos\frac{\pi}{12} = \sin\left(\frac{\pi/4}{2}\right) + \cos\left(\frac{\pi/6}{2}\right) =$

$= \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} + \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} + \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} =$

$= \sqrt{\frac{2-\sqrt{2}}{4}} + \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2} + \frac{\sqrt{2+\sqrt{3}}}{2}$

b) $\sin 72° \cos 18° + \cos 72° \sin 18° = \sin(72° + 18°) = \sin 90° = 1$

a. $\frac{\sin(\alpha + 45°) - \cos(\alpha + 45°)}{\sin(\alpha + 135°) + \cos(\alpha + 315°)}$;

$$= \frac{\sin\alpha\cos 45° + \sin 45°\cos\alpha - [\cos\alpha\cos 45° - \sin\alpha\sin 45°]}{\sin\alpha\cos 135° + \sin 135°\cos\alpha + \cos\alpha\cos 315° - \sin\alpha\sin 315°} =$$

$$= \frac{\frac{\sqrt{2}}{2}\sin\alpha + \cancel{\frac{\sqrt{2}}{2}\cos\alpha} - \cancel{\frac{\sqrt{2}}{2}\cos\alpha} + \frac{\sqrt{2}}{2}\sin\alpha}{\cancel{-\frac{\sqrt{2}}{2}\sin\alpha} + \frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\cos\alpha + \cancel{\frac{\sqrt{2}}{2}\sin\alpha}} =$$

$$= \frac{\cancel{\sqrt{2}}\sin\alpha}{\cancel{\sqrt{2}}\cos\alpha} = \tan\alpha$$

3 Sapendo che $\sin\alpha = \frac{3}{5}$ e $\cos\beta = \frac{5}{13}$, con $\frac{\pi}{2} < \alpha < \pi$ e $0 < \beta < \frac{\pi}{2}$, calcola: $\cos(\alpha+\beta)$, $\sin(\alpha-\beta)$, $\tan 2\alpha$.

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$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta =$$

$$= -\sqrt{1-\sin^2\alpha}\,\cos\beta - \sin\alpha\sqrt{1-\cos^2\beta} =$$

$$= -\sqrt{1-\frac{9}{25}}\cdot\frac{5}{13} - \frac{3}{5}\sqrt{1-\frac{25}{169}} = -\frac{4}{5}\cdot\frac{5}{13} - \frac{3}{5}\cdot\frac{12}{13} =$$

$$= \frac{-20-36}{65} = -\frac{56}{65}$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta = \dots = \frac{63}{65}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2} = \frac{-\frac{3}{2}}{1-\frac{9}{16}} = \frac{-\frac{3}{2}}{\frac{7}{16}} = -\frac{3}{\cancel{2}_{1}}\cdot\frac{\cancel{16}^{8}}{7} = -\frac{24}{7}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$$

4

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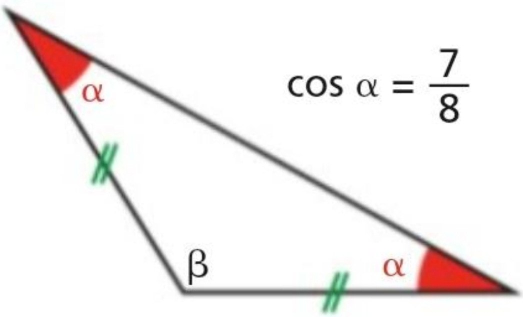


$\cos\alpha = \frac{7}{8}$

Trova $\cos\beta$ e $\sin\alpha$.

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{49}{64}} =$$

$$= \sqrt{\frac{15}{64}} = \frac{\sqrt{15}}{8}$$

$$\beta = \pi - 2\alpha$$

$$\cos\beta = \cos(\pi-2\alpha) =$$

$$= -\cos 2\alpha = -(2\cos^2\alpha - 1) =$$

$$= -2\left(\frac{7}{8}\right)^2 + 1 = -\cancel{2}\frac{49}{\cancel{64}_{32}} + 1 =$$

$$= 1 - \frac{49}{32} = \frac{32-49}{32} = -\frac{17}{32}$$

5 ____/20 Traccia il grafico della funzione

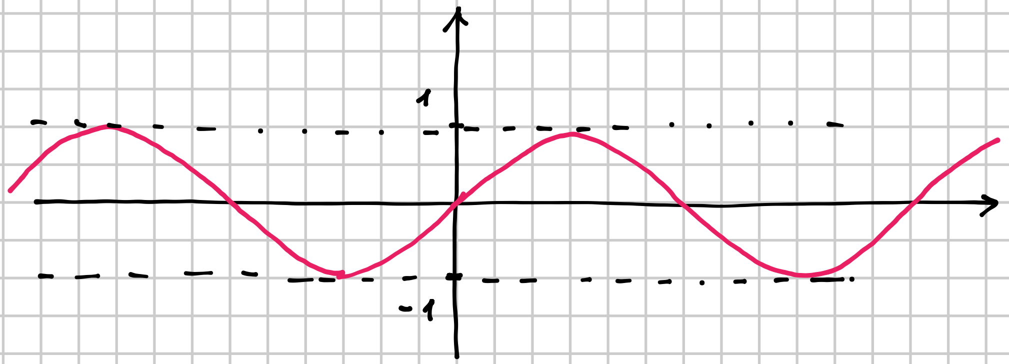
$$y = \frac{\sin 2x}{4\cos x} - 2.$$

$\cos x \neq 0$

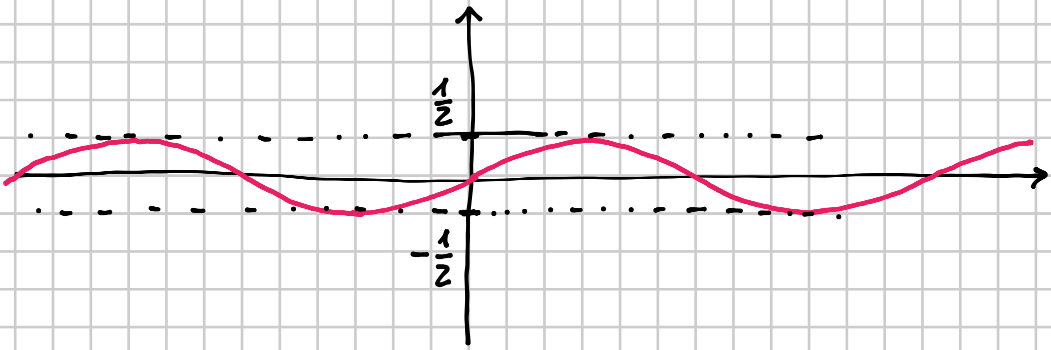
$x \neq \frac{\pi}{2} + k\pi$

$$y = \frac{2\sin x \cancel{\cos x}}{4\cancel{\cos x}} - 2$$

$$\begin{cases} y = \frac{1}{2}\sin x - 2 \\ x \neq \frac{\pi}{2} + k\pi \end{cases}$$

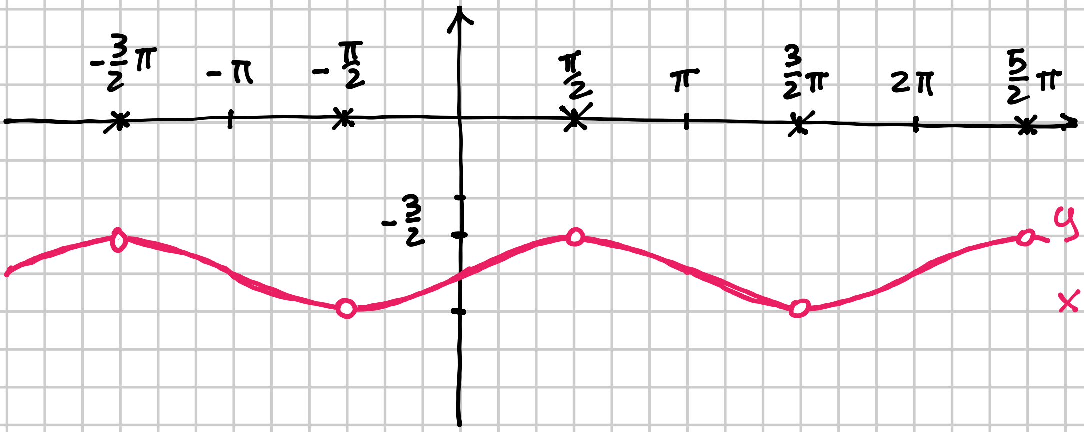


$y = \sin x$



$y = \frac{1}{2}\sin x$

$\Downarrow$ LA "COMPRIMO" $\Uparrow$ DI UN FATTORE $\frac{1}{2}$



$y = \frac{1}{2}\sin x - 2$

$x \neq \frac{\pi}{2} + k\pi$

2 ..../10

Verifica le identità: **a.** $\tan(45° + \alpha) = \dfrac{1 + \sin 2\alpha}{\cos 2\alpha}$; **b.** $2\sin\alpha\cos 2\alpha + \sin\alpha = \sin 3\alpha$.

$$\tan(45° + \alpha) = \frac{1 + \sin 2\alpha}{\cos 2\alpha}$$

$$\frac{\tan 45° + \tan\alpha}{1 - \tan 45° \tan\alpha} = \frac{1 + 2\sin\alpha\cos\alpha}{\cos 2\alpha}$$

$$\frac{1 + \frac{\sin\alpha}{\cos\alpha}}{1 - \frac{\sin\alpha}{\cos\alpha}} = \frac{1 + 2\sin\alpha\cos\alpha}{\cos 2\alpha}$$

$$\frac{\frac{\cos\alpha + \sin\alpha}{\cancel{\cos\alpha}}}{\frac{\cos\alpha - \sin\alpha}{\cancel{\cos\alpha}}} = \frac{1 + 2\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha}$$

$$\frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{\cos^2\alpha + \sin^2\alpha + 2\sin\alpha\cos\alpha}{(\cos\alpha - \sin\alpha)(\cos\alpha + \sin\alpha)}$$

$$\frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{(\cos\alpha + \sin\alpha)^{\cancel{2}}}{(\cos\alpha - \sin\alpha)\cancel{(\cos\alpha + \sin\alpha)}}$$

$$\frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} = \frac{\cos\alpha + \sin\alpha}{\cos\alpha - \sin\alpha} \quad \text{ok!}$$