184 $2\cos^2 x - 3\cos x + 1 = 2\sin^2 x$ $\left[2k\pi; \pm\arccos\left(-\frac{1}{4}\right) + 2k\pi\right]$

$$2\cos^2 x - 3\cos x + 1 = 2\left(1 - \cos^2 x\right)$$

$$2\cos^2 x - 3\cos x + 1 = 2 - 2\cos^2 x$$

$$4\cos^2 x - 3\cos x - 1 = 0 \qquad \Delta = 9 + 16 = 25$$

$$\cos x = \frac{3 \pm 5}{8} = \begin{cases} 1 \\ -\frac{2}{8} = -\frac{1}{4} \end{cases}$$

$\cos x = 1 \Rightarrow x = 2k\pi$ $\qquad$ $\cos x = -\frac{1}{4} \Rightarrow x = \pm\arccos\left(-\frac{1}{4}\right) + 2k\pi$

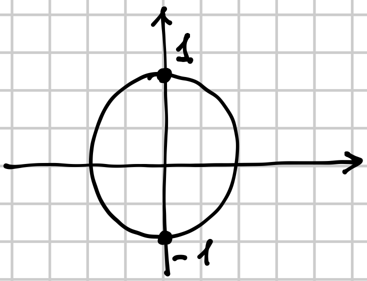
$$\boxed{x = 2k\pi \;\vee\; x = \pm\arccos\left(-\frac{1}{4}\right) + 2k\pi}$$

181 $4\sin^4 x - 5\sin^2 x + 1 = 0$ $\left[\frac{\pi}{2} + k\pi; \pm\frac{\pi}{6} + k\pi\right]$

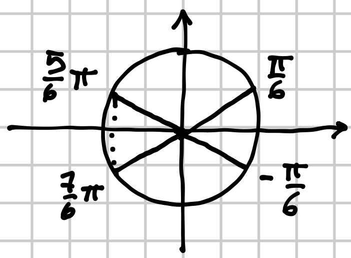
$$\Delta = 25 - 16 = 9$$

$$\sin^2 x = \frac{5 \pm 3}{8} = \begin{cases} 1 \Rightarrow \sin^2 x = 1 \Rightarrow \sin x = \pm 1 \\ \frac{2}{8} = \frac{1}{4} \Rightarrow \sin^2 x = \frac{1}{4} \Rightarrow \sin x = \pm\frac{1}{2} \end{cases}$$

$\sin x = \pm 1$



$x = \frac{\pi}{2} + k\pi$



$\pi - \frac{\pi}{6}$

$\sin x = \frac{1}{2}$ $\qquad$ $x = \frac{\pi}{6} + 2k\pi \;\vee\; x = \frac{5}{6}\pi + 2k\pi$

$\sin x = -\frac{1}{2}$ $\qquad$ $x = -\frac{\pi}{6} + 2k\pi \;\vee\; x = \frac{7}{6}\pi + 2k\pi$

$\pi - \left(-\frac{\pi}{6}\right)$

$$\boxed{x = \frac{\pi}{2} + k\pi \;\vee\; x = \pm\frac{\pi}{6} + 2k\pi \;\vee\; x = \pm\frac{5}{6}\pi + 2k\pi}$$

EQUAZIONI LINEARI IN SENO E COSENO

251 $\sqrt{3}\sin x + \cos x = \sqrt{3}$

$a \sin x + b \cos x = c$

$a, b \neq 0$

$c \neq 0$

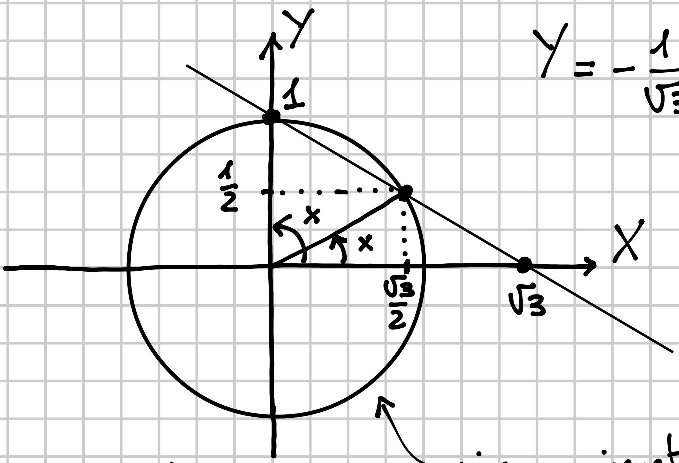
METODO GRAFICO

$$\begin{cases} Y = \sin x \\ X = \cos x \end{cases}$$

$\sqrt{3}\,Y + X = \sqrt{3}$ eq. di una retta

$Y = -\frac{1}{\sqrt{3}}X + 1$

X	Y
0	1
$\sqrt{3}$	0



circ. goniometrico $X^2 + Y^2 = 1$

$\cos^2 x + \sin^2 x = 1$

$$\begin{cases} \sqrt{3}\,Y + X = \sqrt{3} \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} X = -\sqrt{3}\,Y + \sqrt{3} \\ (-\sqrt{3}\,Y + \sqrt{3})^2 + Y^2 = 1 \end{cases}$$

$3Y^2 + 3 - 6Y + Y^2 - 1 = 0 \qquad 4Y^2 - 6Y + 2 = 0$

$2Y^2 - 3Y + 1 = 0$

$\Delta = 9 - 8 = 1$

$Y = \frac{3 \pm 1}{4} = \begin{cases} \frac{1}{2} \\ 1 \end{cases}$

$$\begin{cases} X = 0 \\ Y = 1 \end{cases} \vee \begin{cases} X = \frac{\sqrt{3}}{2} \\ Y = \frac{1}{2} \end{cases}$$

$$\begin{cases} \cos x = 0 \\ \sin x = 1 \end{cases} \vee \begin{cases} \cos x = \frac{\sqrt{3}}{2} \\ \sin x = \frac{1}{2} \end{cases}$$

$$\boxed{x = \frac{\pi}{2} + 2k\pi \quad \vee \quad x = \frac{\pi}{6} + 2k\pi}$$

257 $\sin x + (\sqrt{2} - 1)\cos x - 1 = 0$

$$\begin{cases} Y = \sin x \\ X = \cos x \end{cases}$$

$$\begin{cases} Y + (\sqrt{2}-1)X - 1 = 0 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} Y = -(\sqrt{2}-1)X + 1 \\ X^2 + \left[-(\sqrt{2}-1)X + 1\right]^2 = 1 \end{cases}$$

$$X^2 + (\sqrt{2}-1)^2X^2 \cancel{+1} - 2(\sqrt{2}-1)X = \cancel{1}$$

$$X^2\left[1 + (\sqrt{2}-1)^2\right] - 2(\sqrt{2}-1)X = 0$$

$$X^2\left[1+2+1-2\sqrt{2}\right] - (2\sqrt{2}-2)X = 0$$

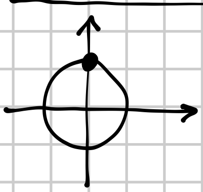
$$X^2\left[4-2\sqrt{2}\right] - (2\sqrt{2}-2)X = 0$$

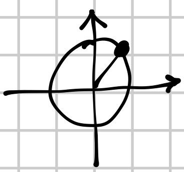
$$X\left[X(4-2\sqrt{2}) - (2\sqrt{2}-2)\right] = 0$$

$$\begin{cases} X = 0 \\ Y = 1 \end{cases} \vee \begin{cases} X = \dfrac{2\sqrt{2}-2}{4-2\sqrt{2}} = \dfrac{\cancel{2}(\sqrt{2}-1)}{-\cancel{2}(\sqrt{2}-2)} \cdot \dfrac{\sqrt{2}+2}{\sqrt{2}+2} = \dfrac{\cancel{2}+2\sqrt{2}-\sqrt{2}-\cancel{2}}{-(2-4)} = \dfrac{\sqrt{2}}{2} \\ Y = -(\sqrt{2}-1)\cdot\dfrac{\sqrt{2}}{2} + 1 = \dfrac{-2+\sqrt{2}}{2} + 1 = \dfrac{\cancel{-2}+\sqrt{2}\cancel{+2}}{2} = \dfrac{\sqrt{2}}{2} \end{cases}$$

$$\begin{cases} \cos x = 0 \\ \sin x = 1 \end{cases} \vee \begin{cases} \cos x = \dfrac{\sqrt{2}}{2} \\ \sin x = \dfrac{\sqrt{2}}{2} \end{cases}$$

$$\boxed{x = \frac{\pi}{2} + 2k\pi \quad \vee \quad x = \frac{\pi}{4} + 2k\pi}$$





257 $\sin x + (\sqrt{2} - 1)\cos x - 1 = 0$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad t = \tan\frac{x}{2} \qquad x \neq \pi + 2k\pi$$

CONTROLLO SE $x = \pi$ È SOLUZIONE

$$\sin\pi + (\sqrt{2}-1)\cos\pi - 1 = 0$$

$$0 + (\sqrt{2}-1)(-1) - 1 = 0$$

$$-\sqrt{2} + \cancel{1} - \cancel{1} = 0$$ FALSO $\Rightarrow$ π non è soluzione

$$\frac{2t}{1+t^2} + (\sqrt{2}-1)\frac{1-t^2}{1+t^2} - 1 = 0$$

$$\frac{2t + \sqrt{2} - \sqrt{2}t^2 - 1 + \cancel{t^2}^{\,2} - 1 - \cancel{t^2}^{\,2}}{\cancel{1+t^2}} = 0$$

$$-\sqrt{2}t^2 + 2t + \sqrt{2} - 2 = 0 \Rightarrow \sqrt{2}t^2 - 2t - \sqrt{2} + 2 = 0$$

$$\frac{\Delta}{4} = 1 - \sqrt{2}(-\sqrt{2}+2) = 1 + 2 - 2\sqrt{2} = (\sqrt{2}-1)^2$$

$$t = \frac{1 \pm (\sqrt{2}-1)}{\sqrt{2}} = \begin{cases} \dfrac{1-\sqrt{2}+1}{\sqrt{2}} = \dfrac{2-\sqrt{2}}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}-2}{2} = \sqrt{2}-1 \\ \dfrac{1+\sqrt{2}-1}{\sqrt{2}} = 1 \end{cases}$$

$$\tan\frac{x}{2} = \sqrt{2}-1 \Rightarrow \frac{x}{2} = \frac{\pi}{8} + k\pi \Rightarrow x = \frac{\pi}{4} + 2k\pi$$

$\vee$

$$\tan\frac{x}{2} = 1 \Rightarrow \frac{x}{2} = \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{2} + 2k\pi$$

EQUAZIONI OMOGENEE DI 2° GRADO

282 $3\sin^2 x + 2\sqrt{3}\sin x\cos x + \cos^2 x = 0$

$a\sin^2 x + b\sin x\cos x + c\cos^2 x = 0 \qquad a,b,c \neq 0$

DIVIDO PER $\cos^2 x$ (è sempre $\neq 0$, altrimenti sarebbe anche $\sin x = 0$ ASSURDO)

$$a\frac{\sin^2 x}{\cos^2 x} + b\frac{\sin x\cancel{\cos x}}{\cos^{\cancel{2}} x} + c\frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 0$$

$$a\cdot\tan^2 x + b\tan x + c = 0$$

$$3\frac{\sin^2 x}{\cos^2 x} + 2\sqrt{3}\frac{\sin x\cancel{\cos x}}{\cos^{\cancel{2}} x} + \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 0$$

$$3\tan^2 x + 2\sqrt{3}\tan x + 1 = 0 \qquad \frac{\Delta}{4} = 3 - 3 = 0$$

$$\left(\sqrt{3}\tan x + 1\right)^2 = 0$$

$$\tan x = -\frac{1}{\sqrt{3}} \Rightarrow \tan x = -\frac{\sqrt{3}}{3} \qquad \boxed{x = -\frac{\pi}{6} + k\pi}$$

286 $2\sqrt{3}\cos^2 x - 2\sin x\cos x = \sqrt{3}\cdot\overbrace{(\sin^2 x + \cos^2 x)}^{1}$

$$2\sqrt{3}\cos^2 x - 2\sin x\cos x - \sqrt{3}\sin^2 x - \sqrt{3}\cos^2 x = 0$$

$$\sqrt{3}\cos^2 x - 2\sin x\cos x - \sqrt{3}\sin^2 x = 0$$

$$-\sqrt{3}\sin^2 x - 2\sin x\cos x + \sqrt{3}\cos^2 x = 0$$

$$\sqrt{3}\frac{\sin^2 x}{\cos^2 x} + 2\frac{\sin x\cos x}{\cos^2 x} - \sqrt{3}\frac{\cos^2 x}{\cos^2 x} = 0$$

$$\sqrt{3}\tan^2 x + 2\tan x - \sqrt{3} = 0 \qquad \frac{\Delta}{4} = 1 + 3 = 4$$

$$\tan x = \frac{-1 \pm 2}{\sqrt{3}} = \begin{cases} -\frac{3}{\sqrt{3}} = -\sqrt{3} \\ \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \end{cases}$$

$$\boxed{x = -\frac{\pi}{3} + k\pi \quad \vee \quad x = \frac{\pi}{6} + k\pi}$$

350 $\dfrac{2\sin x\cos x - \cos x}{3\tan^2 x - 1} = 0$ [impossibile]

C.E. $3\tan^2 x - 1 \neq 0 \Rightarrow \tan^2 x \neq \frac{1}{3} \Rightarrow \tan x \neq \pm\frac{\sqrt{3}}{3}$

$$\begin{cases} x \neq \pm\frac{\pi}{6} + k\pi \\ x \neq \frac{\pi}{2} + k\pi \end{cases}$$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \qquad x = \frac{\pi}{2} + k\pi$ NON ACC.

$\sin x = \frac{1}{2} \qquad x = \frac{\pi}{6} + 2k\pi \lor x = \frac{5}{6}\pi + 2k\pi$

NON ACC.

IMPOSSIBILE

383 $4\sin^2 x + \cos 2x - \sqrt{3}\sin 2x = 0$ $\left[\frac{\pi}{6} + k\pi\right]$

$4\sin^2 x + \cos^2 x - \sin^2 x - 2\sqrt{3}\sin x\cos x = 0$

$3\sin^2 x - 2\sqrt{3}\sin x\cos x + \cos^2 x = 0$

$3\tan^2 x - 2\sqrt{3}\tan x + 1 = 0$

$(\sqrt{3}\tan x - 1)^2 = 0 \qquad \tan x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \boxed{x = \frac{\pi}{6} + k\pi}$

416 $\sin x \sin\left(x+\frac{4}{3}\pi\right)=\cos x\cos\left(\frac{5}{3}\pi+x\right)+\frac{\sqrt{3}-1}{2}$

$$\sin x\left[\sin x\cos\frac{4}{3}\pi+\sin\frac{4}{3}\pi\cos x\right]=\cos x\left[\cos\frac{5}{3}\pi\cos x-\sin\frac{5}{3}\pi\sin x\right]+\frac{\sqrt{3}-1}{2}$$

$$\sin x\left[\sin x\cos\left(\pi+\frac{\pi}{3}\right)+\sin\left(\pi+\frac{\pi}{3}\right)\cos x\right]=\cos x\left[\cos\left(2\pi-\frac{\pi}{3}\right)\cos x-\sin\left(2\pi-\frac{\pi}{3}\right)\sin x\right]+\frac{\sqrt{3}-1}{2}$$

$$\sin x\left[\sin x\cdot\left(-\frac{1}{2}\right)-\frac{\sqrt{3}}{2}\cos x\right]=\cos x\left[\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x\right]+\frac{\sqrt{3}-1}{2}$$

$$-\frac{1}{\cancel{2}}\sin^2 x-\frac{\sqrt{3}}{\cancel{2}}\sin x\cos x=\frac{1}{\cancel{2}}\cos^2 x+\frac{\sqrt{3}}{\cancel{2}}\sin x\cos x+\frac{\sqrt{3}-1}{\cancel{2}}$$

$$\cancel{-\sin^2 x-\cos^2 x}-2\sqrt{3}\sin x\cos x-\sqrt{3}+\cancel{1}=0$$

$$-2\cancel{\sqrt{3}}\sin x\cos x=\cancel{\sqrt{3}}$$

$$-\sin 2x=1$$

$$\sin 2x=-1$$

$$2x=\frac{3}{2}\pi+2k\pi$$

$$\boxed{x=\frac{3}{4}\pi+k\pi}$$