558 $3\tan^2 x + \sqrt{3}\,\tan x \le 0$ $\left[\frac{5}{6}\pi + k\pi \le x \le \pi + k\pi\right]$

$x \neq \frac{\pi}{2} + k\pi$

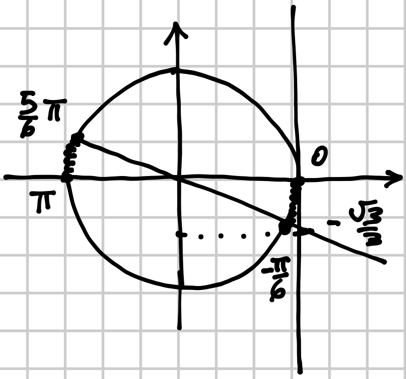
$\tan x = y \Rightarrow \quad 3y^2 + \sqrt{3}\,y \le 0$

$y(3y+\sqrt{3}) = 0 \begin{cases} \nearrow y = 0 \\ \searrow y = -\frac{\sqrt{3}}{3} \end{cases}$

$-\frac{\sqrt{3}}{3} \le y \le 0$

$-\frac{\sqrt{3}}{3} \le \tan x \le 0$

$$\boxed{-\frac{\pi}{6} + k\pi \le x \le k\pi}$$



563 $\cos^2 x - 2\cos x + 1 > 0$ $[x \neq 2k\pi]$

$(\cos x - 1)^2 > 0 \Rightarrow \cos x \neq 1 \Rightarrow x \neq 2k\pi$

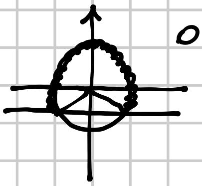
581 $\frac{2\sin x + 1}{\cot x - 1} > 0$ $\left[0 < x < \frac{\pi}{4} \vee \pi < x < \frac{7}{6}\pi \vee \frac{5}{4}\pi < x < \frac{11}{6}\pi\right]$

N]

D]

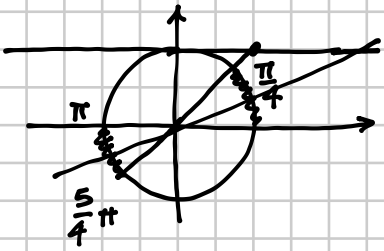
$x \in [0, 2\pi]$ $x \neq 0,\ x \neq \pi$ (C.E. della cotangente)

N] $N > 0$ $2\sin x + 1 > 0$ $\sin x > -\frac{1}{2}$

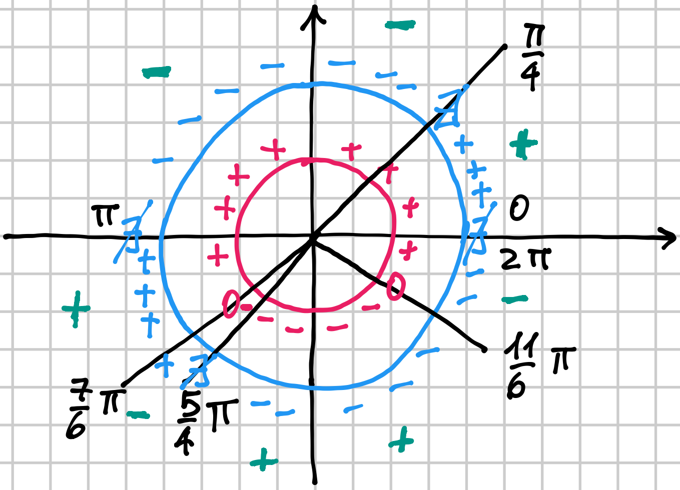


$0 < x < \frac{7}{6}\pi$
$\vee$
$\frac{11}{6}\pi < x < 2\pi$

D] $D > 0$ $\cot x - 1 > 0$ $\cot x > 1$



$0 < x < \frac{\pi}{4}$
$\vee$
$\pi < x < \frac{5}{4}\pi$



$$\boxed{0 < x < \frac{\pi}{4} \quad \vee \quad \pi < x < \frac{7}{6}\pi \quad \vee \quad \frac{5}{4}\pi < x < \frac{11}{6}\pi}$$

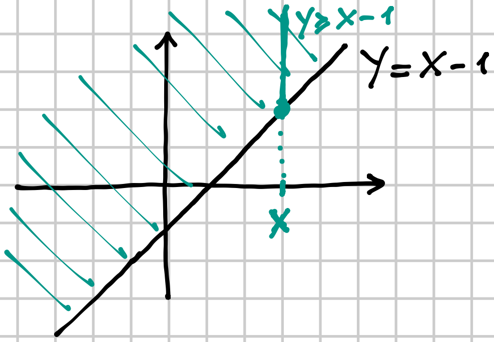
574 $\sin x - \cos x + 1 \geq 0$

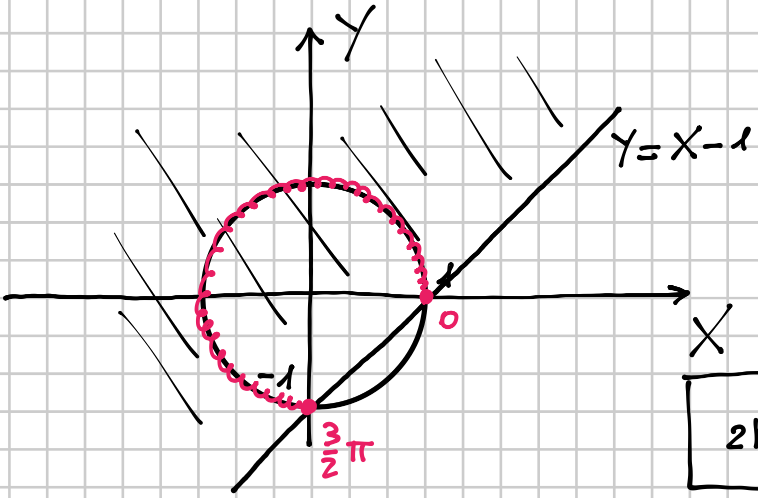
$\left[2k\pi \leq x \leq \frac{3}{2}\pi + 2k\pi\right]$

$Y = \sin x \quad X = \cos x$

$$\begin{cases} Y - X + 1 \geq 0 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} Y \geq X - 1 \\ X^2 + Y^2 = 1 \end{cases}$$

← SEMIPIANO SUPERIORE DI BORDO $Y = X - 1$





$$\boxed{2k\pi \leq x \leq \frac{3}{2}\pi + 2k\pi}$$

576 $\sqrt{3}\sin x - \cos x - 1 \geq 0$ $\left[\frac{\pi}{3} + 2k\pi \leq x \leq \pi + 2k\pi\right]$

$$\begin{cases} \sqrt{3}Y - X - 1 \geq 0 \\ X^2 + Y^2 = 1 \end{cases} \qquad \begin{cases} Y \geq \frac{1}{\sqrt{3}}X + \frac{1}{\sqrt{3}} \\ X^2 + Y^2 = 1 \end{cases}$$

$$\begin{cases} Y \geq \frac{\sqrt{3}}{3}X + \frac{\sqrt{3}}{3} \\ X^2 + Y^2 = 1 \end{cases}$$

Troviamo le intersezioni tra la retta $Y = \frac{\sqrt{3}}{3}X + \frac{\sqrt{3}}{3}$ e la circonferenza (per fare il disegno)

$$\begin{cases} Y = \frac{\sqrt{3}}{3}X + \frac{\sqrt{3}}{3} \\ X^2 + Y^2 = 1 \end{cases}$$

$$X^2 + \left(\frac{\sqrt{3}}{3}X + \frac{\sqrt{3}}{3}\right)^2 = 1$$

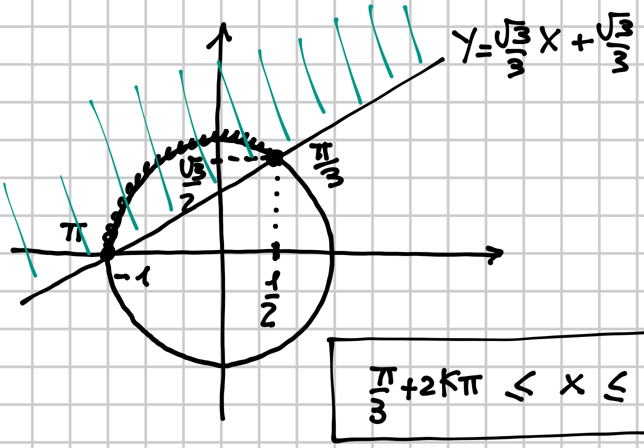
$$X^2 + \frac{1}{3}X^2 + \frac{1}{3} + \frac{2}{3}X = 1$$

$$\frac{4}{3}X^2 + \frac{2}{3}X - \frac{2}{3} = 0$$

$$2X^2 + X - 1 = 0$$

$$X = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = \begin{matrix} -1 \\ \frac{1}{2} \end{matrix}$$

$$\begin{cases} X = -1 \\ Y = 0 \end{cases} \qquad \begin{cases} X = \frac{1}{2} \\ Y = \frac{\sqrt{3}}{2} \end{cases}$$



$$\boxed{\frac{\pi}{3} + 2k\pi \leq x \leq \pi + 2k\pi}$$

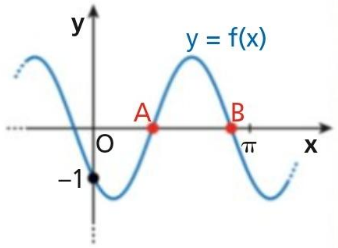
94 Nella figura è rappresentato il grafico della funzione

$f(x) = 2\sin^2 x - 2\cos x \sin x + k$.

a. Determina il valore di k.

b. Trova il periodo della funzione e determina le coordinate di A e B.

c. Dimostra che può essere scritta nella forma $f(x) = A\sin(2x + \varphi)$ e calcola per quali valori di x si ha $f(x) < -1$ in $[0; \pi]$.



$\left[\text{a) } -1;\ \text{b) } T = \pi;\ A\left(\frac{3}{8}\pi; 0\right), B\left(\frac{7}{8}\pi; 0\right);\ \text{c) } 0 < x < \frac{\pi}{4}\right]$

a) $f(x) = 2\sin^2 x - 2\cos x \sin x + k$ $\qquad f: \mathbb{R} \to \mathbb{R}$

$f(0) = -1 \Rightarrow f(0) = 2 \cdot \sin^2 0 - 2\cos 0 \cdot \sin 0 + k = -1 \Rightarrow \boxed{k = -1}$

$f(x) = 2\sin^2 x - 2\cos x \sin x - 1$

b) $f(x) = \underbrace{2\sin^2 x}_{\text{periodo } T_1 = \pi} - \underbrace{\sin 2x}_{\text{periodo } T_2 = \frac{2\pi}{2} = \pi} - 1 \Rightarrow \boxed{T = \pi}$

$A(x_A, 0) \Rightarrow$ $\quad 2\sin^2 x - \sin 2x - 1 = 0$

$2\sin^2 x - 2\sin x\cos x - \cos^2 x - \sin^2 x = 0$

$\sin^2 x - 2\sin x \cos x - \cos^2 x = 0$ (OMOGENEA DI 2° GRADO)

↓ divido per $\cos^2 x$

$\tan^2 x - 2\tan x - 1 = 0$

$\tan x = 1 \pm \sqrt{1+1} = 1 \pm \sqrt{2}$

$\tan x = 1 + \sqrt{2} \Rightarrow x = \frac{3}{8}\pi + k\pi \Rightarrow x_A = \frac{3}{8}\pi$

$\vee$

$\tan x = 1 - \sqrt{2} \Rightarrow x = -\frac{\pi}{8} + k\pi \Rightarrow x_B = -\frac{\pi}{8} + \pi = \frac{7}{8}\pi$

$$\boxed{A\left(\tfrac{3}{8}\pi, 0\right) \quad B\left(\tfrac{7}{8}\pi, 0\right)}$$

c) $f(x) = 2\sin^2 x - \sin 2x - 1 = A\sin(2x+\varphi)$

$$= A\left[\sin 2x\cos\varphi + \sin\varphi\cos 2x\right] =$$

$$= A\sin 2x\cos\varphi + A\sin\varphi\cos 2x =$$

$$= A\cos\varphi\sin 2x + A\sin\varphi\left(1 - 2\sin^2 x\right) =$$

$$= -2A\sin\varphi\sin^2 x + A\cos\varphi\sin 2x + A\sin\varphi$$

$-2A\sin\varphi = 2$ $\qquad \underbrace{A\cos\varphi = -1 \qquad A\sin\varphi = -1}$

$\Downarrow$

$\sin\varphi = -\frac{1}{A}$ $\qquad \cos\varphi = \sin\varphi \Rightarrow \varphi = \frac{\pi}{4} \vee \varphi = \frac{5}{4}\pi$

$\uparrow$

$\sin\varphi = \cos\varphi = \frac{\sqrt{2}}{2}$ oppure $\sin\varphi = \cos\varphi = -\frac{\sqrt{2}}{2}$

1ª SOLUZIONE

$\sin\varphi = -\frac{\sqrt{2}}{2} \Rightarrow -\frac{1}{A} = -\frac{\sqrt{2}}{2} \Rightarrow A = \sqrt{2}$ e $\varphi = \frac{5}{4}\pi$

2ª SOLUZIONE

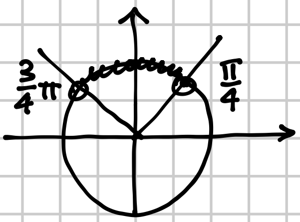
$\sin\varphi = \frac{\sqrt{2}}{2} \Rightarrow -\frac{1}{A} = \frac{\sqrt{2}}{2} \Rightarrow A = -\sqrt{2}$ e $\varphi = \frac{\pi}{4}$

$$f(x) = -\sqrt{2}\sin\left(2x + \frac{\pi}{4}\right)$$

Risolvere

$-\sqrt{2}\sin\left(2x + \frac{\pi}{4}\right) < -1$ $\qquad x \in [0, \pi] \Rightarrow 2x \in [0, 2\pi]$

$\sin\left(2x + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$



$\frac{\pi}{4} < 2x + \frac{\pi}{4} < \frac{3}{4}\pi$

$0 < 2x < \frac{\pi}{2}$ $\qquad \boxed{0 < x < \frac{\pi}{4}}$