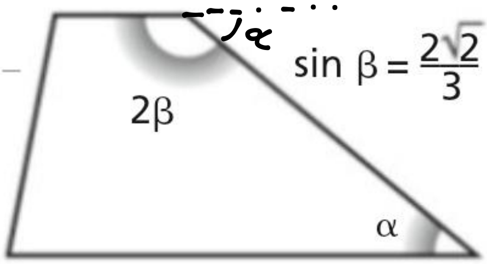
75



Calcola $\cot\alpha$.

$\left[\frac{7\sqrt{2}}{8}\right]$

$$2\beta + \alpha = \pi$$

$$\alpha = \pi - 2\beta$$

$$\sin\alpha = \sin(\pi - 2\beta) =$$

$$= \sin(2\beta) =$$

$$= 2\sin\beta\cos\beta =$$

$$= 2\,\frac{2\sqrt{2}}{3}\sqrt{1-\left(\frac{2\sqrt{2}}{3}\right)^2} =$$

$$= \frac{4\sqrt{2}}{3}\sqrt{1-\frac{8}{9}} = \frac{4\sqrt{2}}{3}\cdot\frac{1}{3} = \frac{4\sqrt{2}}{9}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\left(\frac{4\sqrt{2}}{9}\right)^2} = \sqrt{1-\frac{32}{81}} = \sqrt{\frac{49}{81}} = \frac{7}{9}$$

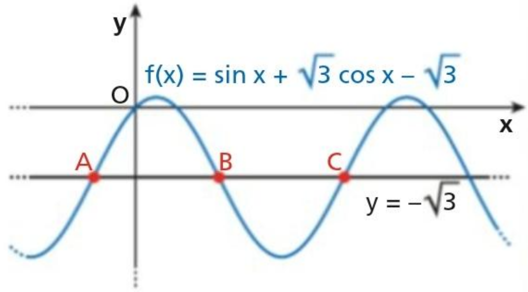
$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{7}{\cancel{9}}}{\frac{4\sqrt{2}}{\cancel{9}}} = \frac{7}{4\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{7\sqrt{2}}{8}}$$

95 Data la funzione rappresentata nella figura:

a. trova le coordinate di A, B e C;

b. determina gli intervalli in cui è positiva;

c. disegna il grafico della funzione $g(x) = 2\cos\left(x + \frac{\pi}{3}\right) + 1$ e determina, graficamente e algebricamente, i punti di intersezione tra i grafici di $f(x)$ e $g(x)$.



[a) $A\left(-\frac{\pi}{3}; -\sqrt{3}\right)$, $B\left(\frac{2}{3}\pi; -\sqrt{3}\right)$, $C\left(\frac{5}{3}\pi; -\sqrt{3}\right)$; b) $2k\pi < x < \frac{\pi}{3} + 2k\pi$; c) $\left(\frac{\pi}{3} + 2k\pi; 0\right)$, $\left(\frac{\pi}{2} + 2k\pi; 1 - \sqrt{3}\right)$]

$f: \mathbb{R} \to \mathbb{R}$

a) $f(x) = -\sqrt{3}$ $\qquad \sin x + \sqrt{3}\cos x \cancel{-\sqrt{3}} = \cancel{-\sqrt{3}}$

$$\sin x + \sqrt{3}\cos x = 0$$

divido per $\cos x$ che è certamente $\neq 0$ (perché altrimenti anche $\sin x$ sarebbe 0)

$$\frac{\sin x}{\cos x} + \sqrt{3}\,\frac{\cos x}{\cos x} = 0$$

$$\tan x = -\sqrt{3}$$

$$x = -\frac{\pi}{3} + k\pi \qquad k = 0 \qquad k = 1 \qquad k = 2$$

$$x = -\frac{\pi}{3} \qquad x = \frac{2}{3}\pi \qquad x = \frac{5}{3}\pi$$

$$A\left(-\frac{\pi}{3}, -\sqrt{3}\right) \quad B\left(\frac{2}{3}\pi, -\sqrt{3}\right) \quad C\left(\frac{5}{3}\pi, -\sqrt{3}\right)$$

b) $f(x) > 0$ $\qquad \sin x + \sqrt{3}\cos x - \sqrt{3} > 0$

$$\begin{cases} Y + \sqrt{3}X - \sqrt{3} > 0 \\ X^2 + Y^2 = 1 \end{cases}$$

Trovo i punti di intersezione

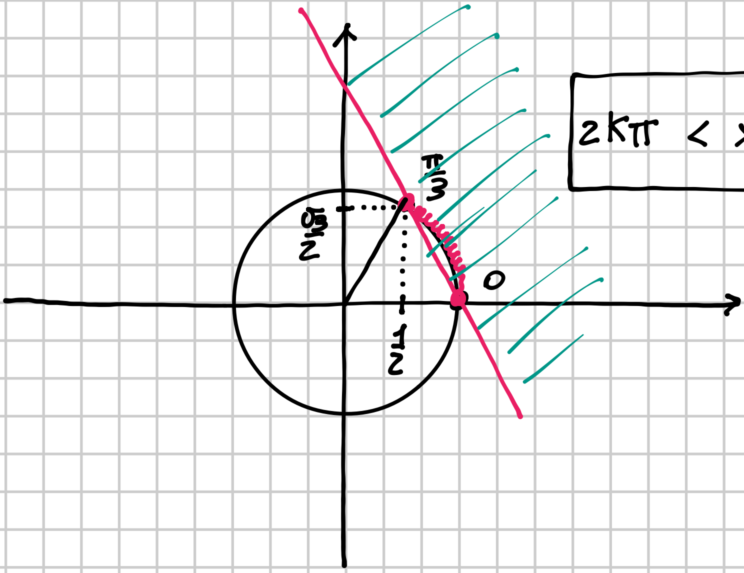
$$\begin{cases} Y = -\sqrt{3}X + \sqrt{3} \\ X^2 + (-\sqrt{3}X + \sqrt{3})^2 - 1 = 0 \end{cases}$$

$$X^2 + 3X^2 + 3 - 6X - 1 = 0$$

$$4X^2 - 6X + 2 = 0$$

$$2X^2 - 3X + 1 = 0 \qquad \Delta = 9 - 8 = 1$$

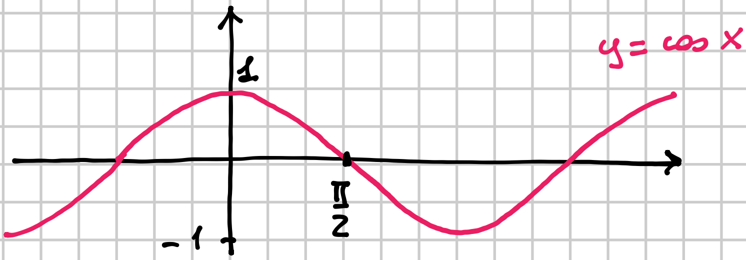
$$X = \frac{3 \pm 1}{4} = \begin{cases} \frac{1}{2} \\ 1 \end{cases} \qquad \begin{cases} X = 1 \\ Y = 0 \end{cases} \vee \begin{cases} X = \frac{1}{2} \\ Y = \frac{\sqrt{3}}{2} \end{cases}$$

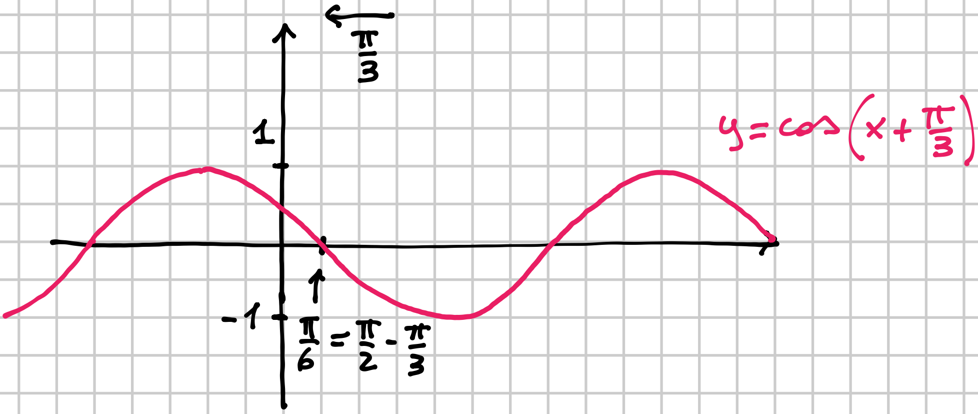


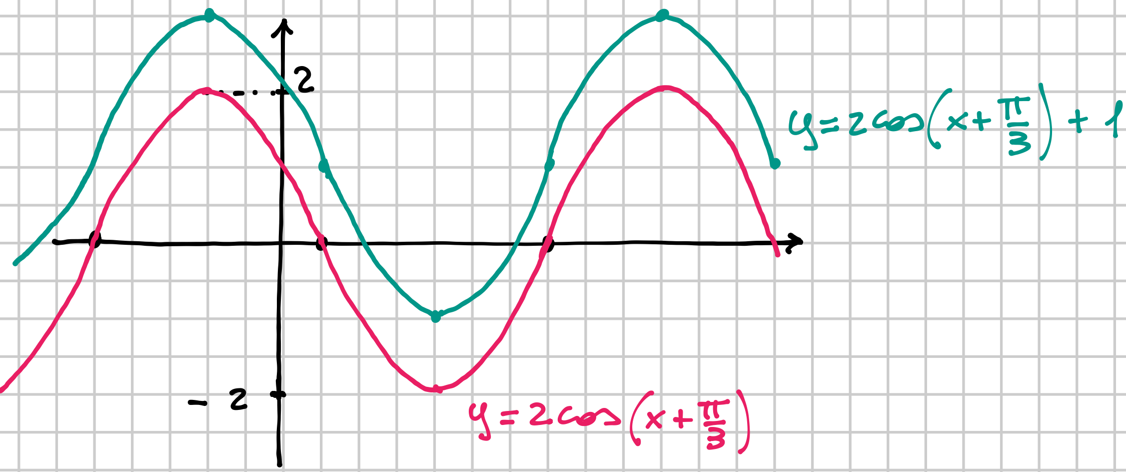
$$\boxed{2k\pi < x < \frac{\pi}{3} + 2k\pi}$$

c) $g(x) = 2\cos\left(x + \frac{\pi}{3}\right) + 1$

$$\cos x \rightarrow \cos\left(x+\frac{\pi}{3}\right) \rightarrow 2\cos\left(x+\frac{\pi}{3}\right)$$
$$\downarrow$$
$$2\cos\left(x+\frac{\pi}{3}\right)+1$$







$$\begin{cases} y = f(x) \\ y = g(x) \end{cases} \qquad \begin{cases} y = \sin x + \sqrt{3}\cos x - \sqrt{3} \\ y = 2\cos\left(x + \frac{\pi}{3}\right) + 1 \end{cases}$$

$$\sin x + \sqrt{3}\cos x - \sqrt{3} = 2\cos\left(x + \frac{\pi}{3}\right) + 1$$

$$\sin x + \sqrt{3}\cos x - \sqrt{3} = 2\left[\cos x \cdot \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}\right] + 1$$

$$\sin x + \sqrt{3}\cos x - \sqrt{3} = \cos x - \sqrt{3}\sin x + 1$$

$$(1+\sqrt{3})\sin x + (\sqrt{3}-1)\cos x - 1 - \sqrt{3} = 0$$

$$\sin x + \frac{\sqrt{3}-1}{\sqrt{3}+1}\cos x - 1 = 0$$

$$\sin x + \frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1}\cos x - 1 = 0$$

$$\sin x + \frac{(\sqrt{3}-1)^2}{2}\cos x - 1 = 0 \quad \text{eq. lineare}$$

$$\begin{cases} Y + \frac{(\sqrt{3}-1)^2}{2} X - 1 = 0 \\ X^2 + Y^2 = 1 \end{cases} \Rightarrow \quad Y = -\frac{4-2\sqrt{3}}{2}X + 1 = (\sqrt{3}-2)X + 1$$

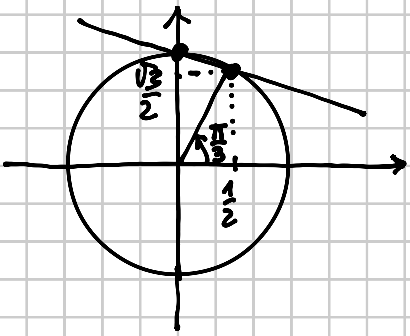
$$X^2 + \left[(\sqrt{3}-2)X + 1\right]^2 = 1$$

$$X^2 + (\sqrt{3}-2)^2X^2 \cancel{+1} + 2(\sqrt{3}-2)X = \cancel{1}$$

$$X^2\left[1 + 3 + 4 - 4\sqrt{3}\right] + 2(\sqrt{3}-2)X = 0$$

$$X^2(8 - 4\sqrt{3}) + 2(\sqrt{3}-2)X = 0$$

$$-4X^2\cancel{(\sqrt{3}-2)} + 2\cancel{(\sqrt{3}-2)}X = 0$$



$$2X^2 - X = 0$$

$$X(2X-1) = 0 \quad \begin{cases} X = 0 \quad Y = 1 \\ X = \frac{1}{2} \quad Y = \frac{\sqrt{3}}{2} \end{cases}$$

$$\begin{cases} X = 0 \\ Y = 1 \end{cases} \Rightarrow x = \frac{\pi}{2} \qquad \begin{cases} X = \frac{1}{2} \\ Y = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow x = \frac{\pi}{3} \qquad x = \frac{\pi}{2} + 2k\pi \ \vee \ x = \frac{\pi}{3} + 2k\pi$$

$$x = \frac{\pi}{2} + 2k\pi \Rightarrow f\left(\frac{\pi}{2} + 2k\pi\right) = \sin\left(\frac{\pi}{2} + 2k\pi\right) + \sqrt{3}\cos\left(\frac{\pi}{2} + 2k\pi\right) - \sqrt{3} =$$

$$= 1 + \sqrt{3} \cdot 0 - \sqrt{3} = 1 - \sqrt{3}$$

$$\boxed{\left(\frac{\pi}{2} + 2k\pi, 1 - \sqrt{3}\right)}$$

$$x = \frac{\pi}{3} + 2k\pi \Rightarrow f\left(\frac{\pi}{3} + 2k\pi\right) = \sin\left(\frac{\pi}{3} + 2k\pi\right) + \sqrt{3}\cos\left(\frac{\pi}{3} + 2k\pi\right) - \sqrt{3} =$$

$$= \frac{\sqrt{3}}{2} + \sqrt{3} \cdot \frac{1}{2} - \sqrt{3} = 0$$

$$\boxed{\left(\frac{\pi}{3} + 2k\pi, 0\right)}$$