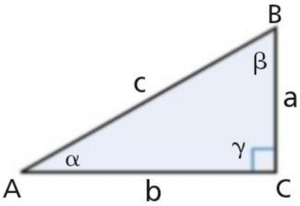
Risolvi il triangolo *ABC*, rettangolo in *C*, noti gli elementi indicati. $\gamma = 90^\circ$

9 $b = 15;$ $\alpha = 30^\circ.$ $[c = 10\sqrt{3}; a = 5\sqrt{3}; \beta = 60^\circ]$

10 $c = 24;$ $\beta = 60^\circ.$ $[b = 12\sqrt{3}; a = 12; \alpha = 30^\circ]$

11 $b = 8;$ $a = 8\sqrt{3}.$ $[c = 16; \beta = 30^\circ; \alpha = 60^\circ]$

12 $c = 48;$ $b = 24.$ $[a = 24\sqrt{3}; \beta = 30^\circ; \alpha = 60^\circ]$



9) $\beta = 90^\circ - \alpha = 90^\circ - 30^\circ = 60^\circ$

$b = c \cdot \cos\alpha \Rightarrow c = \dfrac{b}{\cos\alpha} = \dfrac{15}{\cos 30^\circ} = \dfrac{15}{\frac{\sqrt{3}}{2}} = \dfrac{30}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$

$a = c \cdot \sin\alpha = 10\sqrt{3} \cdot \sin 30^\circ = 10\sqrt{3} \cdot \frac{1}{2} = 5\sqrt{3}$

10) $c = 24$ $\beta = 60^\circ$ $\alpha = 90^\circ - \beta = 30^\circ$

$b = c\cos\alpha = 24 \cdot \cos 30^\circ = 24 \cdot \frac{\sqrt{3}}{2} = 12\sqrt{3}$

$a = c \cdot \sin\alpha = 24 \cdot \sin 30^\circ = 24 \cdot \frac{1}{2} = 12$

11) $b = 8$ $a = 8\sqrt{3}$

$a = b \cdot \tan\alpha \Rightarrow \tan\alpha = \dfrac{a}{b} = \dfrac{\cancel{8}\sqrt{3}}{\cancel{8}} = \sqrt{3} \Rightarrow \alpha = 60^\circ$

$\beta = 90^\circ - \alpha = 30^\circ$

$c = \sqrt{8^2 + (8\sqrt{3})^2} = 8\sqrt{1+3} = 16$

12) $c = 48$ $b = 24$

$b = c\cos\alpha \Rightarrow \cos\alpha = \dfrac{b}{c} = \dfrac{24}{48} = \dfrac{1}{2}$ $\alpha = 60^\circ$

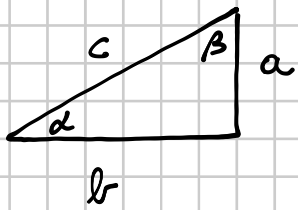
$\beta = 90^\circ - 60^\circ = 30^\circ$

$a = \sqrt{c^2 - b^2} = \sqrt{48^2 - 24^2} = \sqrt{2^2 \cdot 24^2 - 24^2} =$

$= 24\sqrt{4-1} = 24\sqrt{3}$

oppure $a = c \cdot \sin\alpha = 48 \cdot \sin 60^\circ = 48 \cdot \frac{\sqrt{3}}{2} = 24\sqrt{3}$

25 $a = 14$; $\beta = \arccos\frac{2}{3}$. $[c = 21; b \simeq 15{,}6; \gamma \simeq 42°]$



$$b = a \tan\beta = 14 \cdot \tan\left(\arccos\frac{2}{3}\right) =$$

$$= 14 \cdot \frac{\sin\left(\arccos\frac{2}{3}\right)}{\cos\left(\arccos\frac{2}{3}\right)} = 14 \cdot \frac{\sqrt{1-\left(\frac{2}{3}\right)^2}}{\frac{2}{3}} =$$

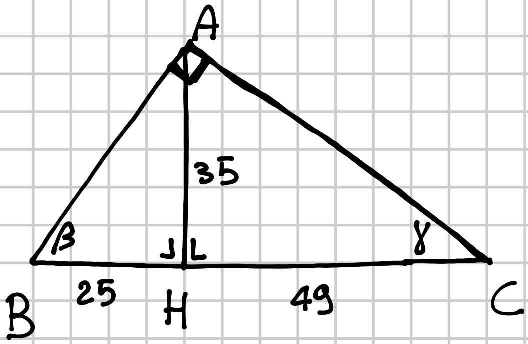
$$= \overset{7}{\cancel{14}} \cdot \frac{3}{\cancel{2}} \sqrt{1-\frac{4}{9}} = 21\sqrt{\frac{9-4}{9}} = \overset{7}{\cancel{21}}\,\frac{\sqrt{5}}{\cancel{3}} = 7\sqrt{5}$$

$$c = \sqrt{a^2+b^2} = \sqrt{14^2+(7\sqrt{5})^2} = \sqrt{7^2 \cdot 2^2 + 7^2 \cdot 5} = 7 \cdot \sqrt{4+5} = 7\sqrt{9} = 7 \cdot 3 = 21$$

oppure $a = c \cdot \cos\beta \Rightarrow c = \frac{a}{\cos\beta} = \frac{14}{\cos\left(\arccos\frac{2}{3}\right)} = \frac{14}{\frac{2}{3}} = \frac{3}{2} \cdot 14 = 21$

$b = c\cos\alpha \Rightarrow \cos\alpha = \frac{b}{c} = \frac{7\sqrt{5}}{21} = \frac{\sqrt{5}}{3}$ $\alpha = \arccos\left(\frac{\sqrt{5}}{3}\right) \simeq 41{,}8°$

86 Nel triangolo rettangolo ABC le proiezioni dei cateti sull'ipotenusa BC sono $BH = 25$ cm e $CH = 49$ cm. Determina i cateti e gli angoli acuti.
$[AB \simeq 43 \text{ cm}; AC \simeq 60{,}2 \text{ cm}; \widehat{B} \simeq 54°; \widehat{C} \simeq 36°]$



$$\overline{BH} : \overline{AH} = \overline{AH} : \overline{HC}$$

$$\overline{AH}^2 = \overline{BH} \cdot \overline{HC}$$

$$\overline{AH} = \sqrt{\overline{BH} \cdot \overline{HC}} = \sqrt{25 \cdot 49} = 5 \cdot 7 = 35$$

$$\overline{AB} = \sqrt{25^2 + 35^2} = \sqrt{5^2 \cdot 5^2 + 5^2 \cdot 7^2} = 5\sqrt{74} \simeq 43$$

$$\overline{AC} = \sqrt{49^2 + 35^2} = \sqrt{7^2 \cdot 7^2 + 7^2 \cdot 5^2} = 7\sqrt{7^2 + 5^2} = 7\sqrt{49 + 25} = 7\sqrt{74} \simeq 60{,}2$$

$$\overline{AH} = \overline{BH} \cdot \tan\beta \Rightarrow \tan\beta = \frac{\overline{AH}}{\overline{BH}}$$

$$\tan\beta = \frac{\overset{7}{\cancel{35}}}{\underset{5}{\cancel{25}}} = \frac{7}{5}$$

$$\beta = \arctan\frac{7}{5} \simeq 54{,}46° \quad \widehat{B}$$

$$\gamma = 90° - \arctan\frac{7}{5} \simeq 35{,}54° \quad \widehat{C}$$