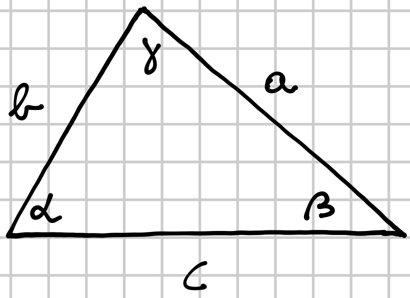
TEOREMA DEL COSENO (DI CARNOT)

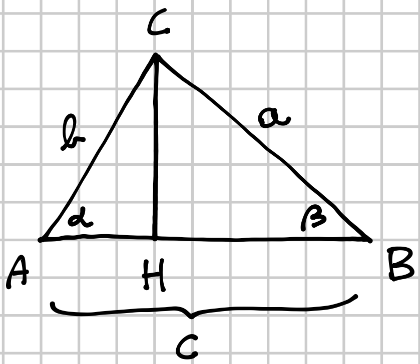


$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

DIMOSTRAZIONE



$$\overline{CB}^2 = (\overline{AB} - \overline{AH})^2 + \overline{HC}^2 = \overline{AB}^2 + \overline{AH}^2 - 2\,\overline{AB}\,\overline{AH} + \overline{HC}^2 =$$

$$= c^2 + (b\cos\alpha)^2 - 2cb\cos\alpha + (b\sin\alpha)^2 =$$

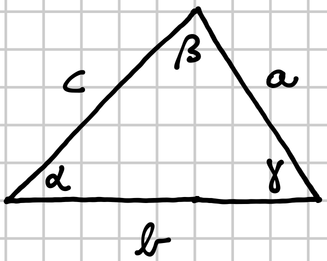
$$= c^2 + b^2\cos^2\alpha - 2bc\cos\alpha + b^2\sin^2\alpha =$$

$$= c^2 + b^2(\overbrace{\cos^2\alpha + \sin^2\alpha}^{1}) - 2bc\cos\alpha$$

$$= c^2 + b^2 - 2bc\cos\alpha \Rightarrow \boxed{a^2 = b^2 + c^2 - 2bc\cos\alpha}$$

Risolvi il triangolo *ABC*, noti gli elementi indicati.

254 $c = 12\sqrt{3}$, $\alpha = \frac{\pi}{4}$, $\gamma = \frac{\pi}{3}$. $\left[\beta = \frac{5}{12}\pi; a = 12\sqrt{2}; b = 6(\sqrt{2}+\sqrt{6})\right]$



$$\beta = \pi - \frac{\pi}{4} - \frac{\pi}{3} = \frac{12-3-4}{12}\pi = \frac{5}{12}\pi$$

TH. SENI $\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$

$$\Rightarrow a = \frac{\sin\alpha}{\sin\gamma}\cdot c = \frac{\frac{\sqrt{2}}{\cancel{2}}}{\frac{\cancel{\sqrt{3}}}{\cancel{2}}}\cdot 12\cancel{\sqrt{3}} = 12\sqrt{2}$$

TH. COSENO $b^2 = a^2 + c^2 - 2ac\cos\beta =$

$$= (12\sqrt{2})^2 + (12\sqrt{3})^2 - 2\cdot 12\sqrt{2}\cdot 12\sqrt{3}\cdot\cos\frac{5}{12}\pi =$$

$$= 12^2\cdot 2 + 12^2\cdot 3 - \cancel{2}\sqrt{6}\cdot 12^2\cdot\frac{\sqrt{6}-\sqrt{2}}{\cancel{4}\,2} =$$

$$= 12^2\cdot\left[5 - 3 + \frac{\sqrt{12}}{2}\right] = 12^2\cdot\left[2 + \frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right] =$$

$$= 12^2\cdot(2+\sqrt{3})$$

$$b = 12\cdot\sqrt{2+\sqrt{3}} =$$

$$= 6\cdot\sqrt{2}\cdot\sqrt{2}\cdot\sqrt{2+\sqrt{3}} =$$

$$= 6\sqrt{2}\cdot\sqrt{4+2\sqrt{3}} =$$

$$= 6\sqrt{2}\cdot\cancel{\sqrt{(1+\sqrt{3})^{2}}} =$$

$$= 6(\sqrt{2}+\sqrt{6})$$