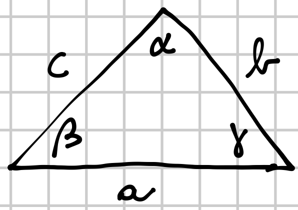
RISOLVERE IL TRIANGOLO

273 $b = 3\sqrt{3}$, $c = 3$, $\beta = \frac{\pi}{3}$. $\left[\alpha = \frac{\pi}{2}; \gamma = \frac{\pi}{6}; a = 6\right]$



$$\frac{c}{\sin\gamma} = \frac{b}{\sin\beta}$$ TH. SENI

$$\Downarrow$$

$$\sin\gamma = \frac{c}{b}\sin\beta = \frac{\cancel{3}}{\cancel{3}\sqrt{3}}\sin\frac{\pi}{3} =$$

$$= \frac{1}{\cancel{\sqrt{3}}}\cdot\frac{\cancel{\sqrt{3}}}{2} = \frac{1}{2}$$

$$\sin\gamma = \frac{1}{2} \Rightarrow \gamma = \frac{\pi}{6} \vee \gamma = \pi - \frac{\pi}{6}$$

$$\gamma = \frac{\pi}{6} \vee \gamma = \frac{5}{6}\pi$$

β γ

$$\frac{\pi}{3} + \frac{5}{6}\pi = \frac{2\pi + 5\pi}{6} = \frac{7}{6}\pi > \pi \Rightarrow \frac{5}{6}\pi$$ NON È ACCETTABILE

$$\Downarrow$$

$$\gamma = \frac{\pi}{6}$$

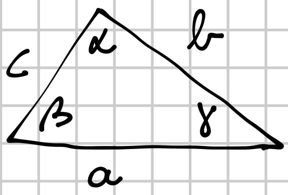
$$\alpha = \pi - \gamma - \beta = \pi - \frac{\pi}{6} - \frac{\pi}{3} = \frac{\pi}{2}$$ (il triangolo è rettangolo e a è l'ipotenusa)

$$a = \sqrt{b^2 + c^2} = \sqrt{(3\sqrt{3})^2 + 3^2} = 3\sqrt{3+1} = 6$$

in alternativa al th. di Pitagora si può usare il th. del coseno o il th. dei seni.

Risolvi il triangolo *ABC*, noti gli elementi indicati.

254 $c = 12\sqrt{3}$, $\alpha = \frac{\pi}{4}$, $\gamma = \frac{\pi}{3}$. $\left[\beta = \frac{5}{12}\pi;\ a = 12\sqrt{2};\ b = 6(\sqrt{2}+\sqrt{6})\right]$



TH. SENI $\Rightarrow$ $\dfrac{c}{\sin\gamma} = \dfrac{a}{\sin\alpha}$

$$a = c\,\frac{\sin\alpha}{\sin\gamma} = 12\sqrt{3}\,\frac{\sin\frac{\pi}{4}}{\sin\frac{\pi}{3}} =$$

$$= 12\cancel{\sqrt{3}}\cdot\frac{\frac{\sqrt{2}}{\cancel{2}}}{\frac{\cancel{\sqrt{3}}}{\cancel{2}}} = 12\sqrt{2}$$

$$\beta = \pi - \alpha - \gamma = \pi - \frac{\pi}{4} - \frac{\pi}{3} = \frac{12-3-4}{12}\pi = \frac{5}{12}\pi$$

TH. COSENO $b^2 = a^2 + c^2 - 2ac\cos\beta =$

$$= 288 + 432 - 2(12\sqrt{2})(12\sqrt{3})\cos\frac{5}{12}\pi =$$

$$= 288 + 432 - \overset{72}{\cancel{288}}\sqrt{6}\cdot\frac{\sqrt{6}-\sqrt{2}}{\cancel{4}} =$$

$$= 720 - 72\cdot 6 + 72\sqrt{12} = 288 + 144\sqrt{3} =$$

$$= 144\left(2+\sqrt{3}\right)$$

$$b = 12\sqrt{2+\sqrt{3}} = 6\sqrt{8+4\sqrt{3}} = (*)$$

$$8 + 4\sqrt{3} = (x+y)^2 = x^2 + y^2 + 2xy$$

$$\begin{cases} x^2 + y^2 = 8 \\ \cancel{2}xy = \overset{2}{\cancel{4}}\sqrt{3} \end{cases} \qquad \begin{cases} x^2 + \dfrac{12}{x^2} = 8 \\ y = \dfrac{2\sqrt{3}}{x} \end{cases} \qquad \begin{cases} x^4 + 12 = 8x^2 \\ y = \dfrac{2\sqrt{3}}{x} \end{cases}$$

$x^4 + 12 = 8x^2 \qquad x^4 - 8x^2 + 12 = 0$

$x^2 = 4 \pm \sqrt{16-12} = 4 \pm 2 = \begin{cases} 6 \\ 2 \end{cases}$

$x = \sqrt{2} \qquad y = \frac{2\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{6}$

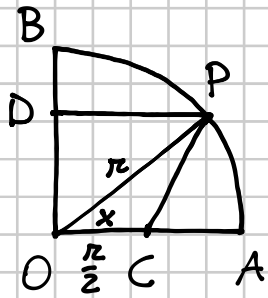
$$\left(\sqrt{2} + \sqrt{6}\right)^2 = 2 + 6 + 2\sqrt{12} = 8 + 4\sqrt{3}$$

quindi $b = 6\sqrt{8 + 4\sqrt{3}} = 6\sqrt{\left(\sqrt{2} + \sqrt{6}\right)^2} = 6\left(\sqrt{2} + \sqrt{6}\right)$

325 In un settore circolare AOB di raggio r e di ampiezza uguale a 90° traccia un raggio OP. Considera la proiezione ortogonale D di P sul raggio OB e il punto medio C del raggio OA. Determina l'angolo $A\widehat{O}P$, sapendo che:

$$\overline{PC}^2 + \overline{PD}^2 = \frac{11}{10} r^2.$$

$$\left[\text{due soluzioni: } \cos A\widehat{O}P = \frac{5 \pm \sqrt{10}}{10}\right]$$



$$0 \le x \le \frac{\pi}{2}$$

$$\overline{PD} = \overline{OP} \cos x = r \cos x$$

$$\overline{PC}^2 = r^2 + \left(\frac{r}{2}\right)^2 - \cancel{2} r \cdot \frac{r}{\cancel{2}} \cos x = r^2 + \frac{r^2}{4} - r^2 \cos x = \frac{5}{4} r^2 - r^2 \cos x$$

$$\overline{PC}^2 + \overline{PD}^2 = \frac{11}{10} r^2$$

$$\frac{5}{4} \cancel{r^2} - \cancel{r^2} \cos x + \cancel{r^2} \cos^2 x = \frac{11}{10} \cancel{r^2} \qquad 0 \le x \le \frac{\pi}{2}$$

$$\cos^2 x - \cos x + \frac{5}{4} - \frac{11}{10} = 0 \qquad \cos^2 x - \cos x + \frac{50-44}{40} = 0$$

$$\cos^2 x - \cos x + \frac{\cancel{6}^{\,3}}{\cancel{40}_{\,20}} = 0 \qquad 20\cos^2 x - 20 \cos x + 3 = 0$$

$$\cos x = \frac{10 \pm \sqrt{100-60}}{20} = \frac{10 \pm \sqrt{40}}{20} =$$

$$= \frac{10 \pm 2\sqrt{10}}{20} = \frac{5 \pm \sqrt{10}}{10}$$

$$x = \arccos\left(\frac{5-\sqrt{10}}{10}\right) \vee x = \arccos\left(\frac{5+\sqrt{10}}{10}\right)$$