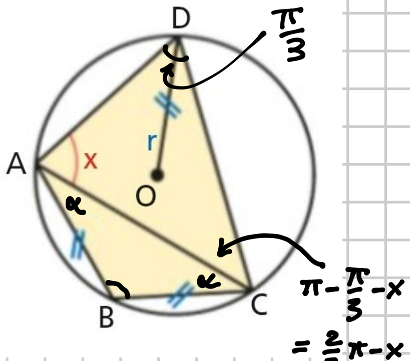
332 La circonferenza della figura ha raggio 2. Calcola l'area $A(x)$ del quadrilatero $ABCD$ e determina per quale valore di x si ha $A(x) = \sqrt{3}$.

$\left[x = 0, x = \frac{2}{3}\pi\right]$



$\overline{BC} = 2r \sin\alpha$

$\cancel{r} = 2\cancel{r}\sin\alpha$

$\sin\alpha = \frac{1}{2}$ $\qquad \alpha = \frac{\pi}{6}$

$\hat{B} = \pi - 2\cdot\frac{\pi}{6} = \pi - \frac{\pi}{3} = \frac{2}{3}\pi$

$\hat{D} = \pi - \frac{2}{3}\pi = \frac{\pi}{3}$

$0 \leq x \leq \frac{2}{3}\pi$

$\mathcal{A}_{ABC} = \frac{1}{2}r^2 \sin\frac{2}{3}\pi = \frac{1}{2}r^2\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}r^2$

$\overline{AC} = 2r\sin\frac{\pi}{3} = 2r\frac{\sqrt{3}}{2} = r\sqrt{3}$ $\qquad \overline{DC} = 2r\sin x$

$\mathcal{A}_{ACD} = \frac{1}{2}\overline{AC}\cdot\overline{DC}\cdot\sin\left(\frac{2}{3}\pi - x\right) = \frac{1}{\cancel{2}}r\sqrt{3}\cdot\cancel{2}r\sin x \sin\left(\frac{2}{3}\pi - x\right) =$

$= \sqrt{3}r^2\cdot\sin x\cdot\left[\sin\frac{2}{3}\pi\cos x - \underbrace{\cos\frac{2}{3}\pi}_{-\frac{1}{2}}\sin x\right] =$

$= \sqrt{3}r^2\sin x\left[\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right] = \frac{3}{2}r^2\sin x\cos x + \frac{\sqrt{3}}{2}r^2\sin^2 x$

$\mathcal{A}_{ABCD} = \mathcal{A}_{ABC} + \mathcal{A}_{ACD} = \frac{\sqrt{3}}{4}r^2 + \frac{3}{2}r^2\sin x\cos x + \frac{\sqrt{3}}{2}r^2\sin^2 x$

$0 \leq x \leq \frac{2}{3}\pi$

$\frac{\sqrt{3}}{4}r^2 + \frac{3}{2}r^2\sin x\cos x + \frac{\sqrt{3}}{2}r^2\sin^2 x = \sqrt{3}$ $\qquad r = 2$

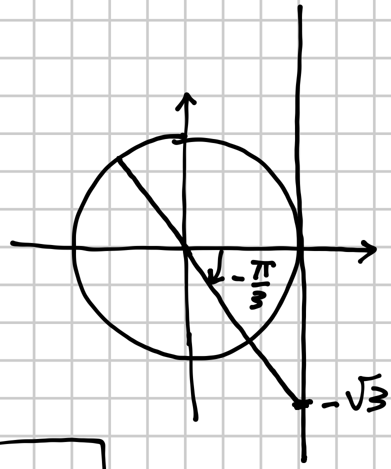
$\cancel{\frac{\sqrt{3}}{4}\cdot 4} + \frac{3}{\cancel{2}}\cdot\overset{2}{\cancel{4}}\sin x\cos x + \frac{\sqrt{3}}{\cancel{2}}\cdot\overset{2}{\cancel{4}}\sin^2 x = \cancel{\sqrt{3}}$

$2\sqrt{3}\sin^2 x + 6\sin x\cos x = 0$ $\qquad 0 \leq x \leq \frac{2}{3}\pi$

$$\cancel{2}\sqrt{3}\sin^2 x + \overset{3}{\cancel{6}}\sin x\cos x = 0 \qquad 0 \le x \le \frac{2}{3}\pi$$

$$\sin x\left(\sqrt{3}\sin x + 3\cos x\right) = 0$$

$$\sin x = 0 \Rightarrow \boxed{x = 0}$$



$$\sqrt{3}\sin x + 3\cos x = 0$$

$$\sqrt{3}\sin x = -3\cos x$$

$$\frac{\sin x}{\cos x} = -\sqrt{3} \Rightarrow \tan x = -\sqrt{3}$$

$$\boxed{x = \pi - \frac{\pi}{3} = \frac{2}{3}\pi}$$