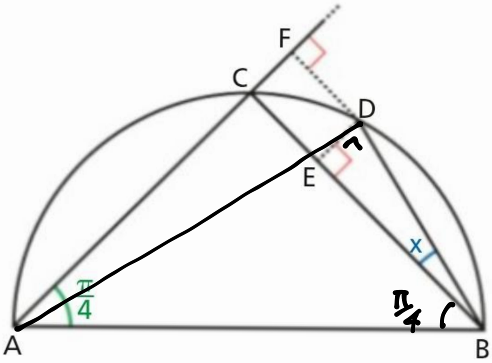
344 Nella semicirconferenza di diametro $\overline{AB} = 2$ della figura, il punto D è un punto qualunque dell'arco $\widehat{BC}$.

a. Determina l'espressione analitica della funzione

$$f(x) = \frac{\overline{DE} + \overline{DF}}{\overline{BC}}.$$

b. Scrivi l'espressione $s(x)$ dell'area del rettangolo $CEDF$ e calcola per quali valori di x si ha $0 < s(x) \leq \frac{1}{4}$.



[a) $y = \sin 2x$, con $0 \leq x \leq \frac{\pi}{4}$; b) $s(x) = \cos 2x - \cos^2 2x, 0 < x < \frac{\pi}{4}$]

a) $0 \leq x \leq \frac{\pi}{4}$ $\qquad \overline{BC} = \sqrt{2}$ $\qquad \overline{DE} = \overline{DB} \cdot \sin x$

$\overline{DF} = \overline{CE} = \overline{BC} - \overline{DB} \cdot \cos x$

$$\overline{DB} = \overline{AB} \cdot \cos\left(\frac{\pi}{4} + x\right) = 2\cos\left(\frac{\pi}{4} + x\right) = 2\left[\cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x\right] =$$

$$= 2\left[\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right] =$$

$$= \sqrt{2}\cos x - \sqrt{2}\sin x$$

$$\overline{DE} = \left(\sqrt{2}\cos x - \sqrt{2}\sin x\right) \cdot \sin x = \sqrt{2}\cos x \sin x - \sqrt{2}\sin^2 x$$

$$\overline{DF} = \sqrt{2} - \left(\sqrt{2}\cos x - \sqrt{2}\sin x\right)\cos x = \sqrt{2} - \sqrt{2}\cos^2 x + \sqrt{2}\sin x\cos x$$

$$f(x) = \frac{\overline{DE} + \overline{DF}}{\overline{BC}} = \frac{\cancel{\sqrt{2}}\cos x\sin x - \cancel{\sqrt{2}}\sin^2 x + \cancel{\sqrt{2}} - \cancel{\sqrt{2}}\cos^2 x + \cancel{\sqrt{2}}\sin x\cos x}{\cancel{\sqrt{2}}} =$$

$$= \cos x\sin x - \cancel{\sin^2 x} + \cancel{1} - \cancel{\cos^2 x} + \sin x\cos x = 2\sin x\cos x =$$

$$= \sin 2x \qquad 0 \leq x \leq \frac{\pi}{4}$$

b) $s(x) = \overline{DE} \cdot \overline{DF} = \left(\sqrt{2}\cos x\sin x - \sqrt{2}\sin^2 x\right)\left(\sqrt{2} - \sqrt{2}\cos^2 x + \sqrt{2}\sin x\cos x\right) =$

$$= 2\cos x\sin x - 2\cos^3 x\sin x + 2\sin^2 x\cos^2 x - 2\sin^2 x + 2\sin^2 x\cos^2 x$$

$$- 2\sin^3 x\cos x$$

$$= 2\cos x \sin x - 2\cos^3 x \sin x + 2\sin^2 x \cos^2 x - 2\sin^2 x + 2\sin^2 x \cos^2 x$$

$$- 2\sin^3 x \cos x =$$

$$= 4\sin^2 x \cos^2 x + \cancel{2\cos x \sin x} - 2\sin^2 x \cancel{\underbrace{- 2\cos^3 x \sin x - 2\sin^3 x \cos x}_{-2\cos x \sin x \underbrace{\left[\cos^2 x + \sin^2 x\right]}_{1}}} =$$

$$= 4\sin^2 x \cos^2 x - 2\sin^2 x = 2\sin^2 x\left(2\cos^2 x - 1\right) =$$

$$= 2\sin^2 x \cdot \cos 2x = -\left(-2\sin^2 x\right)\cos 2x = -\left(-1 + \underbrace{1 - 2\sin^2 x}_{\cos 2x}\right)\cos 2x =$$

$$= -\left(-1 + \cos 2x\right)\cdot \cos 2x = \left(1 - \cos 2x\right)\cdot \cos 2x = \cos 2x - \cos^2 2x$$

Da risolvere $\quad 0 < \cos 2x - \cos^2 2x \leq \frac{1}{4} \qquad 0 \leq x \leq \frac{\pi}{4}$

$\cos 2x = t$

$$\begin{cases} t - t^2 > 0 \\ t - t^2 \leq \frac{1}{4} \end{cases} \qquad \begin{cases} t^2 - t < 0 \\ t^2 - t + \frac{1}{4} \geq 0 \end{cases} \qquad \begin{cases} t(t-1) < 0 \\ (2t-1)^2 \geq 0 \end{cases} \qquad \begin{cases} 0 < t < 1 \\ \forall t \end{cases}$$

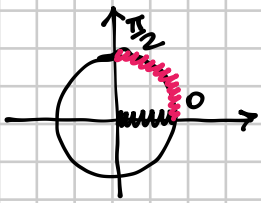
$$4t^2 - 4t + 1 \geq 0$$

$$(2t-1)^2 \geq 0$$

$$0 < t < 1$$

$0 \leq x \leq \frac{\pi}{4}$

$0 \leq x \leq \frac{\pi}{2}$



$0 < \cos 2x < 1 \quad$ con $0 \leq x \leq \frac{\pi}{4}$

$$\Downarrow \quad 0 < 2x < \frac{\pi}{2}$$

$$\boxed{0 < x < \frac{\pi}{4}}$$