62 Dati i due numeri complessi $z_1 = 2a - 1 + 3bi$ e $z_2 = a + b - ai$, determina a e b in modo che:

a. $z_1 = z_2$; **b.** $z_1 = \overline{z_2}$; **c.** z_1 e z_2 siano opposti.

$\left[\text{a) } a = \frac{3}{4}, b = -\frac{1}{4}; \text{ b) } a = \frac{3}{2}, b = \frac{1}{2}; \text{ c) } a = \frac{3}{10}, b = \frac{1}{10}\right]$

$z_1 = 2a - 1 + 3bi$

$z_2 = a + b - ai$

a) $\begin{cases} 2a - 1 = a + b \\ 3b = -a \end{cases}$

$\begin{cases} -6b - 1 = -3b + b \\ a = -3b \end{cases}$

$\begin{cases} -4b = 1 \\ a = -3b \end{cases}$ $\begin{cases} b = -\frac{1}{4} \\ a = \frac{3}{4} \end{cases}$

b) $z_1 = 2a - 1 + 3bi$

$\overline{z_2} = a + b + ai$

$z_1 = \overline{z_2} \Rightarrow \begin{cases} 2a - 1 = a + b \\ 3b = a \end{cases}$ $\begin{cases} 6b - 1 = 3b + b \\ a = 3b \end{cases}$

$\begin{cases} b = \frac{1}{2} \\ a = \frac{3}{2} \end{cases}$

c) $z_1 + z_2 = 0$ $\quad 2a - 1 + 3bi + a + b - ai = 0$

$3a + b - 1 + i(3b - a) = 0$

$\begin{cases} 3a + b - 1 = 0 \\ 3b - a = 0 \end{cases}$ $\begin{cases} 9b + b - 1 = 0 \\ a = 3b \end{cases}$ $\begin{cases} b = \frac{1}{10} \\ a = \frac{3}{10} \end{cases}$

93 $(-5i)^2 - i^{30} + 4i^{20} : (i^6) - i^2 =$ $[-27]$

$= 25i^2 - i^2 + 4i^{14} - (-1) =$

$= 25\cdot(-1) - (-1) + 4i^2 + 1 =$

$= -25 + 1 - 4 + 1 = -27$

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^n = i^r$

r = resto della divisione di n per 4

94 $[2i - 3(-i)^3]^5 + [(-2i)^3]^5 + (3i^7 - 5i)^5 =$ $[-i]$

$= [2i - 3(-i^3)]^5 + [-8i^3]^5 + (3i^3 - 5i)^5 =$

$= [2i - 3i]^5 + [8i]^5 + (-3i - 5i)^5 =$

$= [-i]^5 + 8^5 i^5 + (-8i)^5 = -i^5 + \cancel{8^5 i} - \cancel{8^5 i} = -i^5 = -i$

164 $\dfrac{3+i}{2-i} - \dfrac{i-2}{3-i} + (i-1)(i+2) - i =$ $\left[\dfrac{9i-13}{10}\right]$

$= \dfrac{3+i}{2-i} \cdot \dfrac{2+i}{2+i} - \dfrac{i-2}{3-i} \cdot \dfrac{3+i}{3+i} + i^2 + \cancel{2i} \cancel{-i} - 2 \cancel{-i} =$

$= \dfrac{6+3i+2i+i^2}{4-i^2} - \dfrac{3i+i^2-6-2i}{9-i^2} - 1 - 2 =$

$= \dfrac{6+5i-1}{4+1} - \dfrac{i-1-6}{9+1} - 3 = \dfrac{5+5i}{5} - \dfrac{i-7}{10} - 3 =$

$= \dfrac{10+10i-i+7-30}{10} = \dfrac{-13+9i}{10} = -\dfrac{13}{10} + \dfrac{9}{10}i$

RISOLVERE L'EQUAZIONE

193 $z^2 + |z|^2 - 18 = 0$ [± 3]

$z = x + iy$

$(x+iy)^2 + |x+iy|^2 - 18 = 0$

$x^2 - y^2 + 2xyi + \left(\sqrt{x^2+y^2}\right)^2 - 18 = 0$

$x^2 - \cancel{y^2} + 2xyi + x^2 + \cancel{y^2} - 18 = 0$

$2x^2 - 18 + 2xyi = 0$

$\longrightarrow$ UGUAGLIO A 0 LA PARTE REALE E LA PARTE IMMAGINARIA

$\begin{cases} 2x^2 - 18 = 0 \\ 2xy = 0 \end{cases} \quad \begin{cases} x^2 - 9 = 0 \\ xy = 0 \end{cases} \quad \begin{cases} x = \pm 3 \\ y = 0 \end{cases}$

$\boxed{z = \pm 3}$