SCRIVERE IN FORMA TRIGONOMETRICA

258 $\frac{1}{2}+\frac{\sqrt{3}}{2}i$ $\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right]$

259 $-\sqrt{3}$ $[\sqrt{3}(\cos\pi+i\sin\pi)]$

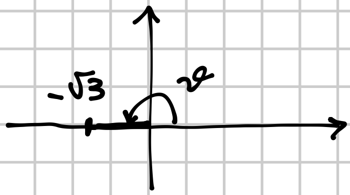
260 1 $[\cos 0+i\sin 0]$

$\boxed{258}$ $z=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ $\qquad \rho=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$

$\tan\vartheta=\frac{y}{x}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}\implies\vartheta=\frac{\pi}{3}$ $\qquad \boxed{z=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$

1° QUADR.

$\boxed{259}$ $z=-\sqrt{3}$

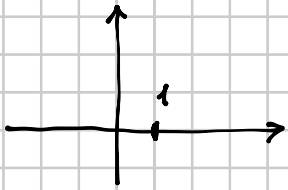


$\rho=\sqrt{3}$

$\vartheta=\pi$

$z=\sqrt{3}\left(\cos\pi+i\sin\pi\right)$

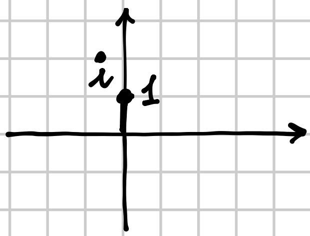
$\boxed{260}$ $z=1$ $\qquad \rho=1$ $\qquad \vartheta=0$



$z=\cos 0+i\sin 0$

Se fosse

$z=i$



$\vartheta=\frac{\pi}{2}$ $\qquad z=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

PRODOTTO DI 2 NUMERI COMPLESSI IN FORMA TRIGONOMETRICA

$$z_1 = \rho_1 (\cos\vartheta_1 + i \sin\vartheta_1)$$

$$z_2 = \rho_2 (\cos\vartheta_2 + i \sin\vartheta_2)$$

$$z_1 \cdot z_2 = \rho_1 \rho_2 (\cos\vartheta_1 + i \sin\vartheta_1)(\cos\vartheta_2 + i \sin\vartheta_2) =$$

$$= \rho_1 \rho_2 (\cos\vartheta_1 \cos\vartheta_2 + i \cos\vartheta_1 \sin\vartheta_2 + i \sin\vartheta_1 \cos\vartheta_2 - \sin\vartheta_1 \sin\vartheta_2)$$

$$= \rho_1 \rho_2 \left[(\cos\vartheta_1 \cos\vartheta_2 - \sin\vartheta_1 \sin\vartheta_2) + i (\sin\vartheta_1 \cos\vartheta_2 + \cos\vartheta_1 \sin\vartheta_2) \right] =$$

$$= \rho_1 \rho_2 \left[\cos(\vartheta_1 + \vartheta_2) + i \sin(\vartheta_1 + \vartheta_2) \right]$$

il modulo del prodotto è il prodotto dei moduli;
l'argomento del prodotto è la <u>SOMMA</u> degli argomenti

In modo analogo:

$$\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2} \left[\cos(\vartheta_1 - \vartheta_2) + i \sin(\vartheta_1 - \vartheta_2) \right]$$

$$z^n = \rho^n \left[\cos(n\vartheta) + i \sin(n\vartheta) \right]$$

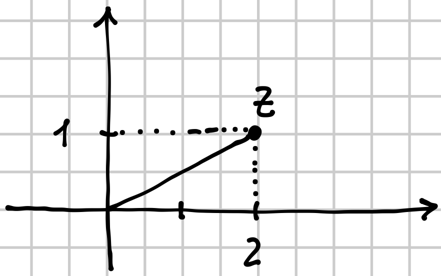
272 $z_1 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$, $\quad z_2 = \frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.

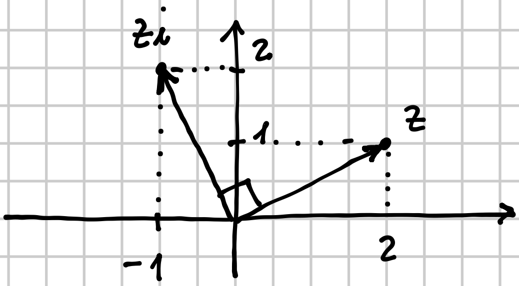
$$z_1 z_2 = 2 \cdot \frac{1}{2}\left(\cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right) + i\ \sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right)\right) =$$

$$= \cos\frac{\pi}{2} + i\ \sin\frac{\pi}{2} = i$$

<u>DOMANDA</u>: Come si interpreta geometricamente la moltiplicazione per i?

ES. $z = 2 + i$ $\qquad z \cdot i = (2+i)\cdot i = 2i + i^2 = -1 + 2i$





<u>RISPOSTA</u>: Significa ruotare il vettore di z di un angolo di $+\frac{\pi}{2}$ (antiorario)

Infatti

$$\left.\begin{array}{l} z = \rho(\cos\vartheta + i\ \sin\vartheta) \\ i = 1\cdot\left(\cos\frac{\pi}{2} + i\ \sin\frac{\pi}{2}\right) \end{array}\right| \Rightarrow zi = \rho\cdot 1\cdot\left(\cos\left(\vartheta+\frac{\pi}{2}\right) + i\ \sin\left(\vartheta+\frac{\pi}{2}\right)\right)$$

274 $z_1 = \sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$, $z_2 = \frac{\sqrt{3}}{2}\left(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi\right)$. $\left[-\frac{3\sqrt{3}}{4} - \frac{3}{4}i\right]$

$$z_1 \cdot z_2 = \sqrt{3} \cdot \frac{\sqrt{3}}{2}\left(\cos\left(\frac{\pi}{3} + \frac{5}{6}\pi\right) + i\sin\left(\frac{\pi}{3} + \frac{5}{6}\pi\right)\right) =$$

$$= \frac{3}{2}\left(\cos\frac{7}{6}\pi + i\sin\frac{7}{6}\pi\right) =$$

$$= \frac{3}{2}\left(-\frac{\sqrt{3}}{2} + i\cdot\left(-\frac{1}{2}\right)\right) = -\frac{3\sqrt{3}}{4} - \frac{3}{4}i$$