341 $\frac{1}{z_1}+\frac{1}{z_2}$; $\quad z_1=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, $\quad z_2=\frac{\sqrt{3}}{3}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$. $\quad \left[\frac{1+\sqrt{3}}{2}-2i\right]$

$$z_1^{-1}=(\sqrt{2})^{-1}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right) \qquad z_2^{-1}=\left(\frac{\sqrt{3}}{3}\right)^{-1}\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$z_1^{-1}+z_2^{-1}=\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)+\frac{3}{\sqrt{3}}\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)=$$

$$=\frac{1}{2}-\frac{1}{2}i+\frac{3}{2\sqrt{3}}-\frac{3}{2}i=\frac{1}{2}+\frac{\cancel{3}}{2\cancel{\sqrt{3}}}\cdot\frac{\sqrt{3}}{\cancel{\sqrt{3}}}-\frac{4}{2}i=$$

$$=\frac{1+\sqrt{3}}{2}-2i$$

SEMPLIFICARE

346 $\frac{(1+i)^4\cdot(\sqrt{3}-i)^3}{(1+i\sqrt{3})^8}$ $\quad \left[\frac{1}{16}(\sqrt{3}-i)\right]$

$$z_1=1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right) \qquad (1+i)^4=(\sqrt{2})^4\left(\cos\frac{4\pi}{4}+i\sin\frac{4\pi}{4}\right)$$

$$|z_1|=\sqrt{1^2+1^2}=\sqrt{2} \qquad \tan\vartheta=1 \qquad \vartheta=\frac{\pi}{4} \qquad =-4$$

$$z_2=\sqrt{3}-i \leftarrow \text{IV QUADR.} \qquad z_2=2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$$

$$|z_2|=\sqrt{3+1}=2 \qquad \tan\vartheta=-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3} \qquad \vartheta=-\frac{\pi}{6}$$

$$(\sqrt{3}-i)^3=2^3\left(\cos\left(-\frac{3\pi}{6}\right)+i\sin\left(-\frac{3\pi}{6}\right)\right)=8(0+i(-1))=-8i$$

$$z_3=1+i\sqrt{3}=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$$

$$|z_3|=\sqrt{1+3}=2 \qquad \tan\vartheta=\sqrt{3} \qquad \vartheta=\frac{\pi}{3}$$

$$(1+i\sqrt{3})^8=2^8\left(\cos\underbrace{\frac{8\pi}{3}}_{\frac{2}{3}\pi+2\pi}+i\sin\frac{8\pi}{3}\right)=2^8\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)=2^7(-1+i\sqrt{3})$$

$$\frac{(1+i)^4(\sqrt{3}-i)^3}{(1+i\sqrt{3})^8} = \frac{-4\cdot(-8i)}{2^7(-1+i\sqrt{3})} = \frac{\cancel{2^5}\, i}{\cancel{2^7}^{\,2}(-1+i\sqrt{3})} =$$

$$= \frac{i}{4(-1+i\sqrt{3})}\cdot\frac{-1-i\sqrt{3}}{-1-i\sqrt{3}} = \frac{-i+\sqrt{3}}{4(1+3)} = \frac{\sqrt{3}-i}{16}$$

Calcola le radici n-esime dell'unità per i seguenti valori di n e rappresentale sulla circonferenza unitaria.

349 $n=6$ $\left[u_0=1, u_1=\frac{1}{2}+i\frac{\sqrt{3}}{2}, u_2=-\frac{1}{2}+i\frac{\sqrt{3}}{2}, u_3=-1, u_4=-\frac{1}{2}-i\frac{\sqrt{3}}{2}, u_5=\frac{1}{2}-i\frac{\sqrt{3}}{2}\right]$

$$z_K = \cos\frac{2K\pi}{n} + i\sin\frac{2K\pi}{n} \qquad K=0,1,2,\dots,n-1$$

$$z_0 = 1$$

$$z_1 = \cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_2 = \cos\frac{4\pi}{6} + i\sin\frac{4\pi}{6} = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_3 = \cos\frac{6\pi}{6} + i\sin\frac{6\pi}{6} = -1$$

$$z_4 = \cos\frac{8\pi}{6} + i\sin\frac{8\pi}{6} = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

$$z_5 = \cos\frac{10\pi}{6} + i\sin\frac{10\pi}{6} = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

RADICI M-ESIME DI UN NUMERO COMPLESSO

Dato $m \in \mathbb{N}$ $(m \geq 2)$ e un numero $z \in \mathbb{C}$, si chiama **RADICE M-ESIMA** di z ciascun numero complesso w tale che

$$w^m = z$$

$$z = \rho(\cos\vartheta + i\sin\vartheta) \qquad w = \bar{\rho}(\cos\bar{\vartheta} + i\sin\bar{\vartheta})$$

$$w^m = z \Rightarrow \bar{\rho}^m\left(\cos(m\bar{\vartheta}) + i\sin(m\bar{\vartheta})\right) = \rho(\cos\vartheta + i\sin\vartheta)$$

Affinché valga questa uguaglianza deve essere

$$\bar{\rho}^m = \rho \Rightarrow \bar{\rho} = \rho^{1/m}$$

$$\begin{cases} \cos(m\bar{\vartheta}) = \cos\vartheta \\ \sin(m\bar{\vartheta}) = \sin\vartheta \end{cases} \qquad m\bar{\vartheta} = \vartheta + 2k\pi \Rightarrow \bar{\vartheta} = \frac{\vartheta + 2k\pi}{m}$$

FORMULA GENERALE

Se $z = \rho(\cos\vartheta + i\sin\vartheta)$

$$\boxed{z_k = \rho^{1/m}\left(\cos\frac{\vartheta + 2k\pi}{m} + i\sin\frac{\vartheta + 2k\pi}{m}\right) \quad k = 0, 1, \dots, m-1}$$

Infatti $z_k^m = \left(\rho^{1/m}\right)^m \cdot \left(\cos\frac{m(\vartheta + 2k\pi)}{m} + i\sin\frac{m(\vartheta + 2k\pi)}{m}\right) = z$

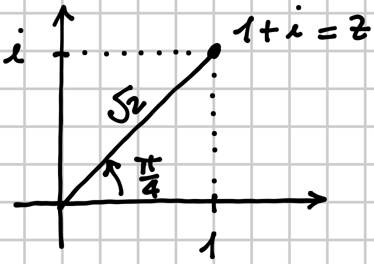
IN PRATICA: trovo la 1ª radice $\to$ $z_0 = \rho^{1/m}\left(\cos\frac{\vartheta}{m} + i\sin\frac{\vartheta}{m}\right)$

$\downarrow$

le altre si trovano in successione facendo ruotare il vettore posizione di $\frac{2\pi}{m}$

ESEMPIO

Calcoliamo le radici 3° (cubiche) di $z = 1 + i$



1° PASSO → trasformo il numero in forma trigonometrica $z = \sqrt{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right)$

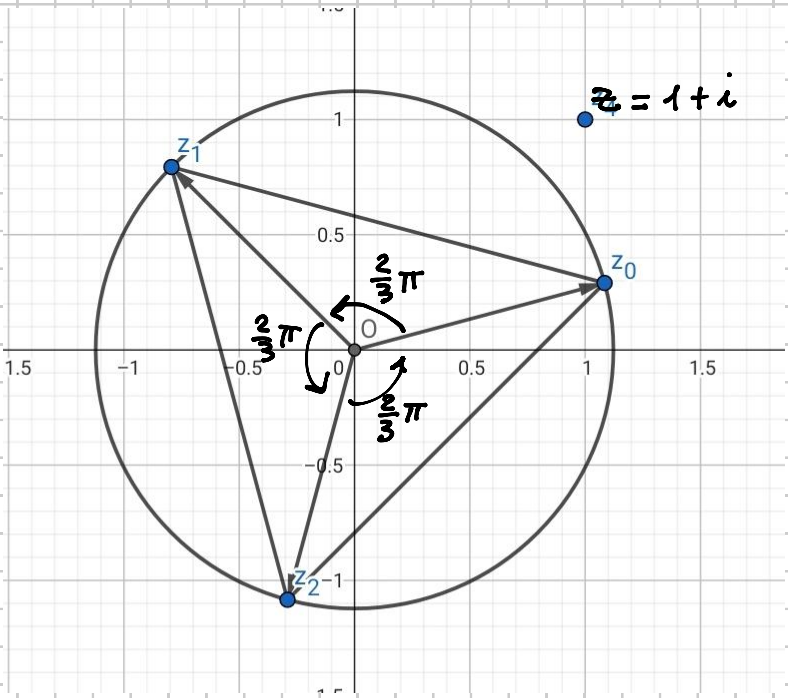
$$z_0 = (\sqrt{2})^{\frac{1}{3}}\left(\cos\frac{\frac{\pi}{4}}{3} + i \sin\frac{\frac{\pi}{4}}{3}\right) = \sqrt[6]{2}\left(\cos\frac{\pi}{12} + i \sin\frac{\pi}{12}\right)$$

$$\frac{\pi}{12} + \frac{2\pi}{3} = \frac{\pi + 8\pi}{12} = \frac{9\pi}{12} = \frac{3}{4}\pi$$

$$z_1 = \sqrt[6]{2}\left(\cos\frac{3}{4}\pi + i \sin\frac{3}{4}\pi\right)$$

$$\frac{3}{4}\pi + \frac{2\pi}{3} = \frac{9\pi + 8\pi}{12} = \frac{17}{12}\pi$$

$$z_2 = \sqrt[6]{2}\left(\cos\frac{17}{12}\pi + i \sin\frac{17}{12}\pi\right)$$



365 $1 + i\sqrt{3}$;

CALCOLARE LE 2 RADICI QUADRATE

$$1+\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\rho = \sqrt{1+3} = 2 \qquad \tan\vartheta = \sqrt{3} \qquad \vartheta = \frac{\pi}{3}$$

$$z_0 = \sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = \sqrt{2}\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \frac{\sqrt{6}}{2} + i\,\frac{\sqrt{2}}{2}$$

$$z_1 = \sqrt{2}\left(\cos\left(\frac{\pi}{6}+\pi\right) + i\sin\left(\frac{\pi}{6}+\pi\right)\right) = \sqrt{2}\left(-\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right) = -\frac{\sqrt{6}}{2} - i\,\frac{\sqrt{2}}{2}$$

Le 2 radici quadrate di un numero z sono sempre numeri opposti tra loro

COSA FAREMO PROSSIMAMENTE?

Risolveremo equazioni che in $\mathbb{R}$ non hanno soluzione

$$x^2 + x + 1 = 0 \qquad \Delta = 1 - 4 = -3 < 0 \Rightarrow \text{in } \mathbb{R} \text{ non ci sono soluzioni}$$

In $\mathbb{C}$:

$$x = \frac{-b \pm r}{2a}$$ dove r è una delle due radici quadrate di Δ

Quali sono le radici quadrate di $\Delta = -3$? $\quad \Delta_1 = \sqrt{3}\,i$ e $\Delta_2 = -\sqrt{3}\,i$

$$\boxed{x = \frac{-1-\sqrt{3}\,i}{2} \quad \vee \quad x = \frac{-1+\sqrt{3}\,i}{2}}$$

Infatti

$$\left(\frac{-1+\sqrt{3}i}{2}\right)^2 + \left(\frac{-1+\sqrt{3}i}{2}\right) + 1 = \frac{1-3-2\sqrt{3}i}{4} + \frac{-1+\sqrt{3}i}{2} + 1 = \frac{-2-2\sqrt{3}i-2+2\sqrt{3}i+4}{4} = 0$$