396 $x^2 - 6i = 0$ $[\pm\sqrt{3}(1+i)]$

$x^2 = 6i$

$x^2 = 6\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$x_1 = \sqrt{6}\left(\cos\frac{\frac{\pi}{2}}{2} + i\sin\frac{\frac{\pi}{2}}{2}\right) = \sqrt{6}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) =$

$= \sqrt{6}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{3}\cdot\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{3}(1+i)$

$x_2 = -\sqrt{3}(1+i)$

$\boxed{x = \pm\sqrt{3}(1+i)}$

398 $x^2 - 2ix + 3 = 0$ $b = -2i \Rightarrow \beta = -i$ $[3i, -i]$

$\frac{\Delta}{4} = \beta^2 - ac = (-i)^2 - 3 = -1 - 3 = -4$

le 2 radici quadrate sono $\pm 2i$

$x = i \pm 2i = \begin{cases} -i \\ 3i \end{cases}$

$\boxed{x = -i \quad \vee \quad x = 3i}$

392 $x^4+23x^2-50=0$ $[\pm\sqrt{2}, \pm 5i]$

$t=x^2$

$t^2+23t-50=0$

$(t+25)(t-2)=0$

$t=-25 \qquad t=2$

$x^2=-25 \qquad x^2=2$

$$\boxed{x=\pm 5i \quad \vee \quad x=\pm\sqrt{2}}$$

IL POLINOMIO SI SCOMPONE

$(x-5i)(x+5i)(x-\sqrt{2})(x+\sqrt{2})$

401 $x^2+\dfrac{(2-i)^2-1+4i}{i}x+3=0$ $[3i, -i]$

$$\frac{(2-i)^2-1+4i}{i}=\frac{4+i^2-\cancel{4i}-1+\cancel{4i}}{i}=\frac{4-1-1}{i}=\frac{2}{i}\cdot\frac{i}{i}=\frac{2i}{-1}=-2i$$

$x^2-2ix+3=0 \qquad \dfrac{\Delta}{4}=(-i)^2-3=-4$

$x=i\pm 2i= \begin{cases} 3i \\ -i \end{cases}$

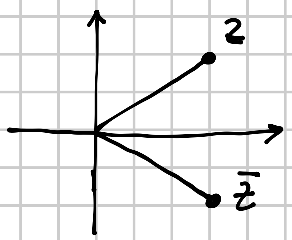
$$\boxed{x=3i \quad \vee \quad x=-i}$$

414 Dato $z \in \mathbb{C}$, sia $\bar{z}$ il suo complesso coniugato. Rappresenta nel piano di Gauss l'insieme $E \cap F$, con:

$$E = \{z \in \mathbb{C}: |z - 1| < |\bar{z}|\}, \quad F = \left\{z \in \mathbb{C}: \left|z - \frac{1}{2}\right| \leq 2\right\}.$$

$$E = \{ z \in \mathbb{C} : |z-1| < |\bar{z}| \}$$

$|\bar{z}| = |z|$



$z = x + iy \qquad |x + iy - 1| < |x + iy|$

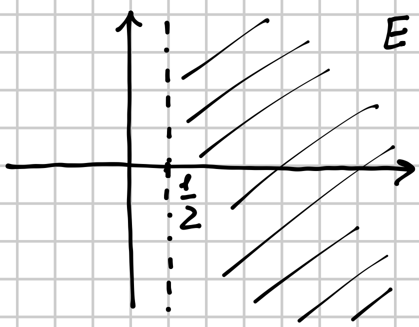
$$|(x-1) + iy| < |x + iy|$$

$$\sqrt{(x-1)^2 + y^2} < \sqrt{x^2 + y^2}$$

$$(x-1)^2 + y^2 < x^2 + y^2$$

$$\cancel{x^2} + 1 - 2x + \cancel{y^2} < \cancel{x^2} + \cancel{y^2}$$

$$-2x < -1 \implies x > \frac{1}{2}$$



$$F = \{ x \in \mathbb{C} \mid |z - \tfrac{1}{2}| \leq 2 \}$$

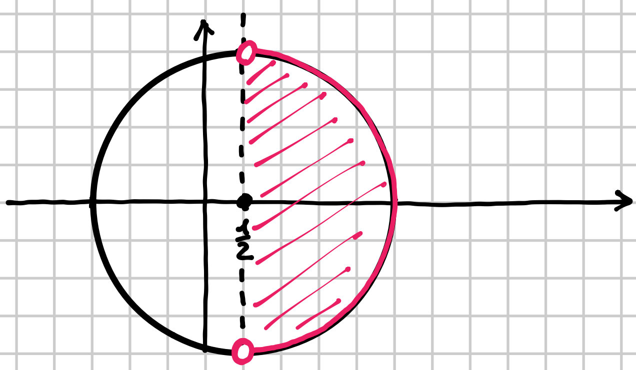
CERCHIO DI CENTRO $(\frac{1}{2}, 0)$ E RAGGIO 2
(BORDO + INTERNO)

$|x + iy - \frac{1}{2}| \leq 2 \qquad |(x - \frac{1}{2}) + iy| \leq 2$

$$\sqrt{(x - \tfrac{1}{2})^2 + y^2} \leq 2$$

$$(x - \tfrac{1}{2})^2 + y^2 \leq 4$$

$(x - \frac{1}{2})^2 + y^2 = 4$
CIRCONF. DI CENTRO $(\frac{1}{2}, 0)$
E RAGGIO 2



411 **a.** Calcola, dopo averne opportunamente semplificato l'espressione, le soluzioni z_1, z_2, z_3 dell'equazione $(1+i)z^3 = 8\sqrt{2}\,i$, con $z \in \mathbb{C}$.

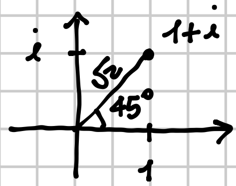
b. Calcola $z_1^2 + z_2^2 + z_3^2$.

[a) $z_1 = 2(\cos 15° + i \cdot \sin 15°)$, $z_2 = 2(\cos 135° + i \cdot \sin 135°)$, $z_3 = 2(\cos 255° + i \cdot \sin 255°)$; b) 0]

a) $(1+i)z^3 = 8\sqrt{2}\,i$

$$z^3 = \frac{8\sqrt{2}i}{1+i} \cdot \frac{1-i}{1-i} = \frac{8\sqrt{2}i - 8\sqrt{2}i^2}{1^2 - i^2} = \frac{8\sqrt{2}i + 8\sqrt{2}}{1-(-1)} =$$

$$= \frac{8\sqrt{2}i + 8\sqrt{2}}{2} = 4\sqrt{2}i + 4\sqrt{2} = 4\sqrt{2}(1+i) =$$



$$= 4\sqrt{2} \cdot \sqrt{2}(\cos 45° + i \sin 45°) =$$

$$= 8(\cos 45° + i \sin 45°)$$

$$z_1 = \sqrt[3]{8}\left(\cos \frac{45°}{3} + i \sin \frac{45°}{3}\right) = 2(\cos 15° + i \sin 15°)$$

le altre radici si trovano aggiungendo $\frac{360°}{3} = 120°$

$$z_2 = 2(\cos 135° + i \sin 135°)$$

$$z_3 = 2(\cos 255° + i \sin 255°)$$

b)

$$z_1^2 + z_2^2 + z_3^2 = 2^2(\cos 30° + i \sin 30°) + 2^2(\cos 270° + i \sin 270°) +$$

$$+ 2^2(\cos \underbrace{510°}_{150°+360°} + i \sin 510°)$$

$$= 4\left[\frac{\sqrt{3}}{2} + \frac{1}{2}i + 0 - i - \frac{\sqrt{3}}{2} + \frac{1}{2}i\right] = 0$$