21 $\left[\sqrt{2}\left(\cos\frac{7}{4}\pi + i\sin\frac{7}{4}\pi\right)\right]^3 + \left[2\sqrt{2}\left(\cos\frac{5}{4}\pi + i\sin\frac{5}{4}\pi\right)\right]^2 =$ $[-2+6i]$

$$= (\sqrt{2})^3\left(\cos\frac{3\cdot 7}{4}\pi + i\sin\frac{3\cdot 7}{4}\pi\right) + (2\sqrt{2})^2\left(\cos\frac{\cancel{2}\cdot 5}{\cancel{4}_2}\pi + i\sin\frac{\cancel{2}\cdot 5}{\cancel{4}_2}\pi\right) =$$

$$= 2\sqrt{2}\left(\cos\frac{21}{4}\pi + i\sin\frac{21}{4}\pi\right) + 8\left(\cos\frac{5}{2}\pi + i\sin\frac{5}{2}\pi\right) =$$

$$\frac{21}{4}\pi = 5\pi + \frac{\pi}{4} = 4\pi + \frac{5}{4}\pi \qquad \frac{5}{2}\pi = 2\pi + \frac{\pi}{2}$$

$$= 2\sqrt{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) + 8(0+i) = -2 - 2i + 8i = \boxed{-2+6i}$$

23 $\sqrt{2}\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right) : \left(\cos\frac{7}{6}\pi + i\sin\frac{7}{6}\pi\right) =$

$$= \sqrt{2}\left(\cos\left(\frac{3}{2}\pi - \frac{7}{6}\pi\right) + i\sin\left(\frac{3}{2}\pi - \frac{7}{6}\pi\right)\right) =$$

$$= \sqrt{2}\left(\cos\frac{9\pi - 7\pi}{6} + i\sin\frac{\pi}{3}\right) = \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = \boxed{\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i}$$

52 $(x^2+4)(x^3-27i) = 0$ $\left[\frac{3}{2}(\sqrt{3}+i), \frac{3}{2}(-\sqrt{3}+i), -3i, \pm 2i\right]$

$$(x^2+4)(x^3-27i) = 0$$

$$x^2+4=0 \quad \vee \quad x^3-27i=0$$

$$x^2+4=0 \qquad x^2=-4 \qquad x=\pm 2i$$

$$x^3-27i=0 \qquad x^3=27i$$

$$\sqrt[3]{27} \qquad x^3 = 27\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$x_0 = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 3\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} + \frac{3}{2}i \qquad \frac{2\pi}{3}$$

$$x_1 = 3\left(\cos\left(\frac{\pi}{6}+\frac{2}{3}\pi\right) + i\sin\left(\frac{\pi}{6}+\frac{2}{3}\pi\right)\right) = 3\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$$

$$\frac{1+4}{6}\pi = \frac{5}{6}\pi$$

$$x_2 = 3\left(\cos\left(\underbrace{\frac{5}{6}\pi+\frac{2}{3}\pi}_{\frac{9}{6}\pi=\frac{3}{2}\pi}\right)+i\sin\left(\frac{5}{6}\pi+\frac{2}{3}\pi\right)\right)=$$

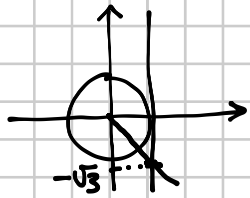
$$=3\left(0+i(-1)\right)=-3i$$

$$\boxed{x=\pm 2i \quad \vee \quad x=-3i \quad \vee \quad x=\pm\frac{3\sqrt{3}}{2}+\frac{3}{2}i}$$

53 $x^3 = 4-4i\sqrt{3}$ ← IV QUADRANTE

$$\rho=\sqrt{4^2+(-4\sqrt{3})^2}=\sqrt{16+48}=\sqrt{64}=8$$

$$\tan\vartheta=\frac{-4\sqrt{3}}{4}=-\sqrt{3} \implies \vartheta=-\frac{\pi}{3}$$



$$x^3=8\left(\cos\left(-\frac{\pi}{3}\right)+i\sin\left(-\frac{\pi}{3}\right)\right)$$

$$x_k=2\left(\cos\left(-\frac{\pi}{9}+\frac{2k\pi}{3}\right)+i\sin\left(-\frac{\pi}{9}+\frac{2k\pi}{3}\right)\right) \qquad k=0,1,2$$

$\sqrt[3]{8}$

Rappresentare nel piano di Gauss

217 $|2z-3|=|z+i|$

$z = x+iy$

$$|2(x+iy)-3| = |x+iy+i|$$

$$|2x+2yi-3| = |x+(y+1)i|$$

$$|(2x-3)+2yi| = |x+(y+1)i|$$

$$\sqrt{(2x-3)^2+(2y)^2} = \sqrt{x^2+(y+1)^2}$$

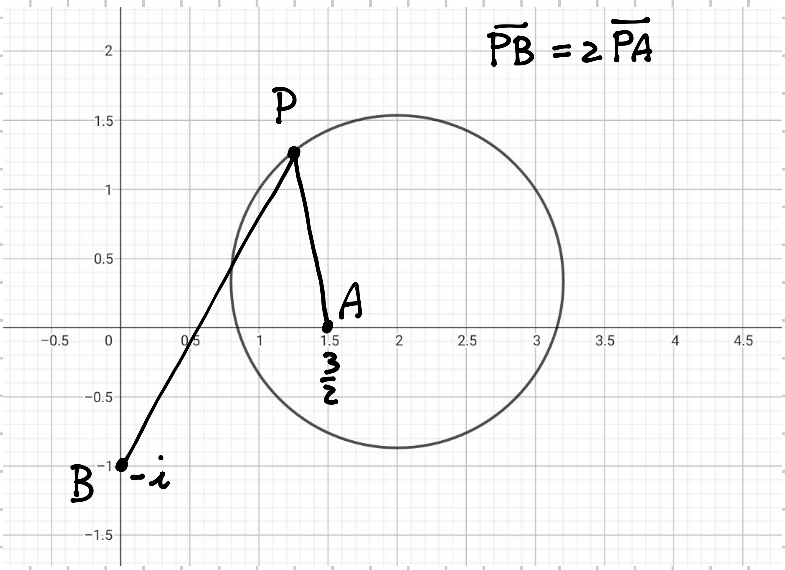
$$4x^2+9-12x+4y^2 = x^2+y^2+1+2y$$

$$3x^2+3y^2-12x-2y+8=0 \qquad \alpha=-\frac{a}{2},\ \beta=-\frac{b}{2}$$

$$x^2+y^2-4x-\frac{2}{3}y+\frac{8}{3}=0 \qquad C\left(2,\frac{1}{3}\right) \qquad r=\sqrt{\alpha^2+\beta^2-c}$$

$$r=\sqrt{4+\frac{1}{9}-\frac{8}{3}}=\sqrt{\frac{36+1-24}{9}}=\frac{\sqrt{13}}{3}$$

Il luogo geometrico dei punti che soddisfano l'uguaglianza data è la circonferenza di centro $C\left(2,\frac{1}{3}\right)$ e raggio $r=\frac{\sqrt{13}}{3}$



$|z+i|=|2z-3|$

$|z-(-i)|=2\left|z-\frac{3}{2}\right|$

luogo dei punti z la cui distanza da $-i$ è il doppio della distanza da $\frac{3}{2}$