290 Verifica che le rette $r: \begin{cases} x+y-6=0 \\ 2x+z-12=0 \end{cases}$ e $s: \begin{cases} 2x+z=0 \\ x+y-3=0 \end{cases}$ sono complanari e parallele, e determina l'equazione del piano che le contiene. $[2x+4y-z-12=0]$

$$r: \begin{cases} 2x+t-12=0 \\ y=6-x \\ z=t \end{cases} \quad \begin{cases} x=6-\frac{1}{2}t \\ y=6-6+\frac{1}{2}t \\ z=t \end{cases} \quad \begin{cases} x=6-\frac{1}{2}t \\ y=\frac{1}{2}t \\ z=t \end{cases} \quad \begin{cases} x=6-t \\ y=t \\ z=2t \end{cases}$$

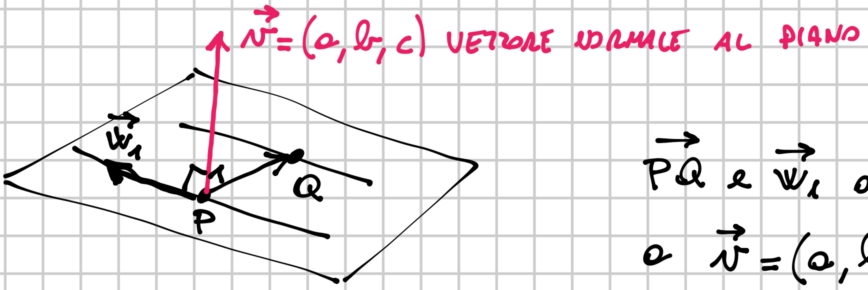
$$\vec{w}_1=(-1,1,2)$$

$$s: \begin{cases} x=-\frac{1}{2}t \\ y=3-x=3+\frac{1}{2}t \\ z=t \end{cases} \quad \begin{cases} x=-t \\ y=3+t \\ z=2t \end{cases} \quad \vec{w}_2=(-1,1,2)$$

SONO L'UNO MULTIPLO SCALARE DELL'ALTRO

r e s sono parallele, quindi complanari

I punti $\underbrace{P(6,0,0)}_{\in r}$ e $\underbrace{Q(0,3,0)}_{\in s}$ appartengono al piano



$$\vec{PQ}=(-6,3,0)$$

$\vec{PQ}$ e $\vec{w}_1$ devono essere perpendicolari a $\vec{v}=(a,b,c)$

$$\begin{cases} \vec{PQ}\cdot\vec{v}=0 \\ \vec{w}_1\cdot\vec{v}=0 \end{cases} \quad \begin{cases} (-6,3,0)\cdot(a,b,c)=-6a+3b=0 \\ (-1,1,2)\cdot(a,b,c)=-a+b+2c=0 \end{cases}$$

$$\begin{cases} -6a+3a-6c=0 \\ b=a-2c \end{cases} \quad \begin{cases} -3a=6c \\ b=a-2c \end{cases} \quad \begin{cases} a=-2c \\ b=-4c \end{cases} \quad \begin{cases} a=2 \\ b=4 \\ c=-1 \end{cases}$$

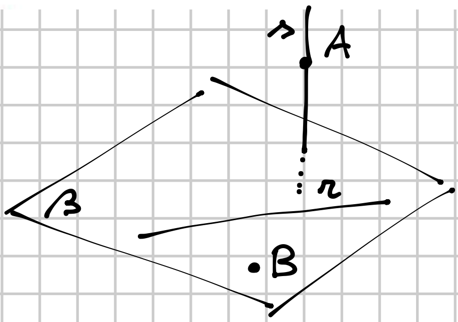
FORMULA PER PIANO CON $\vec{v}=(a,b,c)$ PASSANTE PER $P(x_0,y_0,z_0)$

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0 \qquad P(6,0,0)$$

$$2(x-6)+4y-z=0 \qquad \boxed{2x+4y-z-12=0}$$

291 Scrivi le equazioni cartesiane della retta s passante per il punto $A(1; 2; 3)$ e perpendicolare al piano β, che contiene la retta r di equazioni $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z+1}{1}$ e passa per il punto $B(6; 0; 1)$.

$\left[\frac{x-1}{7} = y-2 = \frac{3-z}{16}\right]$



Scriviamo r in forma parametrica

$$\begin{cases} \frac{x-1}{2} = t \\ \frac{y-3}{2} = t \\ \frac{z+1}{1} = t \end{cases} \qquad \begin{cases} x = 2t+1 \\ y = 2t+3 \\ z = t-1 \end{cases}$$

$$t=1 \quad \begin{cases} x = 3 \\ y = 5 \\ z = 0 \end{cases} \quad P(3,5,0) \qquad t=0 \quad \begin{cases} x = 1 \\ y = 3 \\ z = -1 \end{cases} \quad Q(1,3,-1)$$

$$ax + by + cz + d = 0$$

$$\begin{matrix} P(3,5,0) \\ Q(1,3,-1) \\ B(6,0,1) \end{matrix} \quad \begin{cases} 3a + 5b + d = 0 \\ a + 3b - c + d = 0 \\ 6a + c + d = 0 \end{cases} \quad \begin{cases} 3a + 5b + d = 0 \\ a + 3b + 6a + d + d = 0 \\ c = -6a - d \end{cases} \quad \begin{cases} a = \frac{-5b-d}{3} \\ 7a + 3b + 2d = 0 \\ c = -6a - d \end{cases}$$

$$\begin{cases} a = -\frac{5}{3}b - \frac{1}{3}d \\ 7\left(-\frac{5}{3}b - \frac{1}{3}d\right) + 3b + 2d = 0 \Rightarrow -\frac{35}{3}b - \frac{7}{3}d + 3b + 2d = 0 \\ c = -6a - d \end{cases}$$

$$\Downarrow$$

$$-\frac{26}{\cancel{3}}b - \frac{1}{\cancel{3}}d = 0$$

$$b = -\frac{1}{26}d$$

$$\begin{cases} a = -\frac{5}{3} + \frac{26}{3} = \frac{21}{3} = 7 \\ b = 1 \\ c = -42 + 26 = -16 \\ d = -26 \end{cases}$$

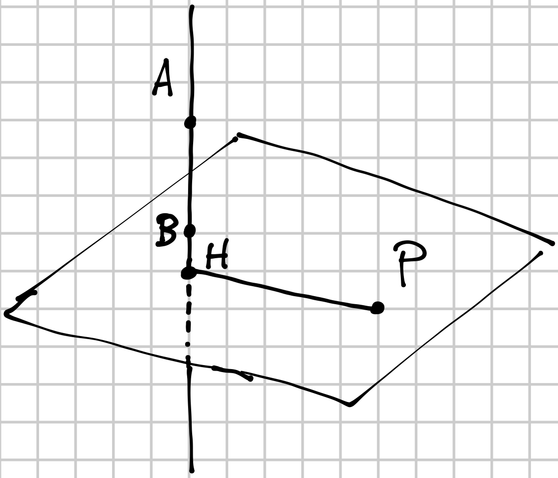
PIANO β: $7x + y - 16z - 26 = 0$

$\vec{n} = (7, 1, -16)$

RETTA s ha $\vec{n}$ come VETTORE DIREZIONE, PASSA PER $A(1,2,3)$

$$s: \begin{cases} x = 1 + 7t \\ y = 2 + t \\ z = 3 - 16t \end{cases} \qquad \boxed{\frac{x-1}{7} = y - 2 = \frac{3-z}{16}}$$

295 Calcola la distanza del punto $P(1;-3;5)$ dalla retta r passante per $A(-1;0;1)$ e $B(1;2;0)$. [5]



trovo la retta AB $\qquad \vec{AB}=(2,2,-1)$

$$\frac{x-x_A}{l}=\frac{y-y_A}{m}=\frac{z-z_A}{n}$$

oppure

$$\frac{x-x_A}{x_B-x_A}=\frac{y-y_A}{y_B-y_A}=\frac{z-z_A}{z_B-z_A}$$

$$\frac{x+1}{2}=\frac{y}{2}=1-z$$

$$\begin{cases} x=-1+2t \\ y=2t \\ z=1-t \end{cases}$$

Trovo il piano perpendicolare ad AB passante per $P(1,-3,5)$

$$2(x-1)+2(y+3)-(z-5)=0$$

$$2x-2+2y+6-z+5=0$$

$$2x+2y-z+9=0$$

Intersezione retta-piano

$$2(-1+2t)+2(2t)-(1-t)+9=0$$

$$-2+4t+4t-1+t+9=0 \qquad 9t=-6 \qquad t=-\frac{2}{3}$$

$$H:\begin{cases} x=-1+2\left(-\frac{2}{3}\right)=-1-\frac{4}{3}=-\frac{7}{3} \\ y=-\frac{4}{3} \\ z=1+\frac{2}{3}=\frac{5}{3} \end{cases}$$

$$H\left(-\frac{7}{3},-\frac{4}{3},\frac{5}{3}\right) \quad P(1,-3,5)$$

$$\overline{HP}=\sqrt{\left(-\frac{7}{3}-1\right)^2+\left(-\frac{4}{3}+3\right)^2+\left(\frac{5}{3}-5\right)^2}=$$

$$=\sqrt{\frac{100}{9}+\frac{25}{9}+\frac{100}{9}}=\sqrt{\frac{225}{9}}=\frac{15}{3}=\boxed{5}$$