VERIFICARE L'IDENTITÀ

153 $$\frac{1}{k+1}\binom{n}{k}+\binom{n+1}{k+1}=\frac{n-k+1}{n+1}\binom{n+2}{k+1}$$

$$\frac{1}{k+1}\cdot\frac{n!}{k!\,(n-k)!}+\frac{(n+1)!}{(k+1)!\,(n+1-(k+1))!}=\frac{n-k+1}{n+1}\;\frac{(n+2)!}{(k+1)!\,(n+2-(k+1))!}$$

$$\frac{n!}{(k+1)!\,(n-k)!}+\frac{(n+1)!}{(k+1)!\,(n\cancel{+1}-k\cancel{-1})!}=\frac{n-k+1}{n+1}\cdot\frac{(n+2)!}{(k+1)!\,(n+2-k-1)!}$$

$$\frac{n!+(n+1)!}{(k+1)!\,(n-k)!}=\frac{\cancel{(n-k+1)}\cdot(n+2)!}{(n+1)\,(k+1)!\,\underbrace{(n-k+1)!}_{\cancel{(n-k+1)}\,(n-k)!}}$$

$$\frac{n!+(n+1)\cdot n!}{(k+1)!\,(n-k)!}=\frac{(n+2)!}{(n+1)\,(k+1)!\,(n-k)!}$$

$$\frac{n!\,[1+n+1]}{(k+1)!\,(n-k)!}=\frac{(n+2)\cancel{(n+1)}\,n!}{\cancel{(n+1)}\,(k+1)!\,(n-k)!}$$

$$\frac{n!\,(n+2)}{(k+1)!\,(n-k)!}=\frac{(n+2)\,n!}{(k+1)!\,(n-k)!}$$

$$6\cdot\binom{x}{x-2}-\binom{x+1}{x-2}=2\cdot\binom{x}{x-4}$$ [7]

C.E. $x-4\geq 0 \Rightarrow x\geq 4 \quad x\in\mathbb{N}$

$$\overset{3}{\cancel{6}}\cdot\frac{x!}{(x-2)!\underbrace{(x-(x-2))!}_{\cancel{2!}}}-\frac{(x+1)!}{(x-2)!\underbrace{(x+1-(x-2))!}_{3!}}=2\cdot\frac{x!}{(x-4)!\,4!}$$

$$\frac{3\cdot x!}{(x-2)!}-\frac{(x+1)!}{(x-2)!\cdot 6}=\cancel{2}\cdot\frac{x!}{(x-4)!\,4\cdot 3\cdot\cancel{2}}$$

$$\frac{3\cdot x\cdot(x-1)\cdot\cancel{(x-2)!}}{\cancel{(x-2)!}}-\frac{(x+1)\cdot x\cdot(x-1)\cdot\cancel{(x-2)!}}{\cancel{(x-2)!}\cdot 6}=\frac{x(x-1)(x-2)(x-3)\cancel{(x-4)!}}{\cancel{(x-4)!}\cdot 12}$$

$$3\cancel{x}\cancel{(x-1)}-\frac{(x+1)\cdot\cancel{x}\cdot\cancel{(x-1)}}{6}=\frac{\cancel{x}\cancel{(x-1)}(x-2)(x-3)}{12}$$

$$3-\frac{x+1}{6}=\frac{(x-2)(x-3)}{12}$$

$$\frac{36-\cancel{2x}-2}{\cancel{12}}=\frac{x^2-3x-\cancel{2x}+6}{\cancel{12}}$$

$$x^2-3x-28=0 \qquad \Delta=9+112=121$$

$$x=\frac{3\pm 11}{2}=\begin{cases}-\frac{8}{2}=-4 & \text{NON ACC.}\\[4pt] \frac{14}{2}=7\end{cases}$$

$$\boxed{x=7}$$

BINOMIO DI NEWTON

277 $(2a^2+3a^3)^4$; $\left(\dfrac{a}{2}+x\right)^8$.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

$$(A+B)^4 = A^4 + 4A^3B + 6A^2B^2 + 4AB^3 + B^4$$

$$(\underbrace{2a^2}_{A}+\underbrace{3a^3}_{B})^4 = (2a^2)^4 + 4(2a^2)^3(3a^3) + 6(2a^2)^2(3a^3)^2 + 4(2a^2)(3a^3)^3 + (3a^3)^4$$

$$= 16a^8 + 96a^9 + 216a^{10} + 216a^{11} + 81a^{12}$$

1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

$$(A+B)^8 = A^8 + 8A^7B + 28A^6B^2 + 56A^5B^3 + 70A^4B^4 + 56A^3B^5$$

$$+ 28A^2B^6 + 8AB^7 + B^8$$

$$\left(\frac{a}{2}+x\right)^8 = \frac{a^8}{256} + 8\frac{a^7}{128}x + 28\frac{a^6}{64}x^2 + 56\frac{a^5}{32}x^3 + 70\frac{a^4}{16}x^4 + 56\frac{a^3}{8}x^5$$

$$+ 28\frac{a^2}{4}x^6 + 8\frac{a}{2}x^7 + x^8 =$$

$$= \frac{a^8}{256} + \frac{a^7x}{16} + \frac{7}{16}a^6x^2 + \frac{7}{4}a^5x^3 + \frac{35}{8}a^4x^4 + 7a^3x^5 + 7a^2x^6 + 4ax^7 + x^8$$