TRASFORMAZIONI DI LORENTZ

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma\left(t - \frac{\beta}{c}x\right) \end{cases}$$

SOSTITUISCO v CON $-v$

INVERSE
$$\begin{cases} x = \gamma(x' + vt') \\ t = \gamma\left(t' + \frac{\beta}{c}x'\right) \end{cases}$$

Proviamo a invertire le equazioni algebricamente

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma\left(t - \frac{\beta}{c}x\right) \end{cases}$$

ricaviamo x e t in funzione di x' e t'

Dalla prima $\frac{x'}{\gamma} = x - vt \qquad x = \frac{x'}{\gamma} + vt$

$$\Downarrow$$

$$t' = \gamma\left(t - \frac{\beta}{c}\left(\frac{x'}{\gamma} + vt\right)\right)$$

$$t' = \gamma t - \frac{\beta}{c}x' - \gamma\frac{\beta}{c}vt \implies t' = \gamma t - \frac{\beta}{c}x' - \gamma\beta^2 t$$

$\frac{v}{c} = \beta$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \gamma^2 = \frac{1}{1-\beta^2} \qquad 1-\beta^2 = \frac{1}{\gamma^2}$$

$$\gamma t - \gamma\beta^2 t = t' + \frac{\beta}{c}x'$$

$$\gamma(1-\beta^2)t = t' + \frac{\beta}{c}x'$$

$$\cancel{\gamma} \cdot \frac{1}{\gamma^{\cancel{2}}} t = t' + \frac{\beta}{c}x' \implies \boxed{t = \gamma\left(t' + \frac{\beta}{c}x'\right)}$$

$$x = \frac{x'}{\gamma} + vt \implies x = \frac{x'}{\gamma} + v\gamma\left(t' + \frac{\beta}{c}x'\right) = \frac{x'}{\gamma} + v\gamma t' + \gamma\frac{\beta}{c}vx' =$$

$\frac{v}{c} = \beta$

$$\frac{1}{\gamma^2} + \beta^2 = \frac{1}{\frac{1}{1-\beta^2}} + \beta^2 = 1 - \beta^2 + \beta^2 = 1$$

$$= \frac{x'}{\gamma} + v\gamma t' + \gamma\beta^2 x' = \gamma\left(\frac{x'}{\gamma^2} + \beta^2 x' + vt'\right) =$$

$$= \gamma\Big(x'\underbrace{\left(\frac{1}{\gamma^2} + \beta^2\right)}_{1} + vt'\Big) \implies \boxed{x = \gamma(x' + vt')}$$

66 ◆◆◆ Nel sistema di riferimento S un punto materiale è nella posizione $x = 40$ m all'istante $t = 0{,}10$ μs. Un secondo sistema di riferimento S' si muove lungo l'asse x nel verso positivo con velocità $v = 2{,}0 \times 10^8$ m/s.

▶ Determina le coordinate dello stesso punto materiale in S'.

[27 m; $1{,}5 \times 10^{-8}$ s]

$$\begin{cases} x' = \gamma(x - vt) \\ t' = \gamma\left(t - \frac{\beta}{c}x\right) \end{cases}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{2{,}0\times10^8\ m/s}{3{,}0\times10^8\ m/s}\right)^2}} = \frac{1}{\sqrt{1-\left(\frac{2}{3}\right)^2}} = \frac{1}{\sqrt{1-\frac{4}{9}}} = \frac{1}{\sqrt{\frac{5}{9}}} = \frac{3}{\sqrt{5}}$$

$$x' = \frac{3}{\sqrt{5}}\left(40\,m - \left(2{,}0\times10^8\,\frac{m}{s}\right)\left(0{,}10\times10^{-6}\,s\right)\right) = 26{,}832\ldots\,m \simeq 27\,m$$

$$t' = \frac{3}{\sqrt{5}}\left(0{,}10\times10^{-6}\,s - \frac{2{,}0\times10^8\,\frac{m}{s}}{\left(3{,}0\times10^8\,\frac{m}{s}\right)^2}\,40\,m\right) =$$

$$= 1{,}4907\ldots\times10^{-8}\,s \simeq \boxed{1{,}5\times10^{-8}\,s}$$